6.3 - Geometric Sequences

A sequence where there is a common ratio, r, between consecutive terms. A new term is generated by multiplying/dividing each term by the same number.

eg. $5, \overset{\times 3}{15}, \overset{\times 3}{45}, 135, \ldots$ r = 3

$$r = \frac{t_2}{t_1}$$

40, 20, 10, 5, 5/2, ... r = $\frac{1}{2}$

3, -6, 12, -24, 48, ... r = -2

Geometric Sequence Formula

$$t_n = ar^{n-1}$$

where a is the first term, and r is the common ratio.

Ex. 1 Find t_7 for each sequence.

a) $t_n = -2(3)^{n-1}$

$t_7 = -2(3)^{7-1}$

$= -2(3)^6$

$= -1458$

b) $t_n = 100\left(\frac{1}{4}\right)^{n-1}$

$t_7 = 100\left(\frac{1}{4}\right)^{7-1}$

$= 100\left(\frac{1}{4}\right)^6$

$= \overset{25}{\cancel{100}}\left(\frac{1}{\underset{1024}{\cancel{4096}}}\right)$

$= \frac{25}{1024}$

.

Ex. 2 Simplify the powers.

a) $3^{x-1} \bullet 3^{x+5}$

$= 3^{x-1+x+5}$

$= 3^{2x+4}$

b) $32^{x+2} \bullet 8^6$

$= (2^5)^{x+2} \cdot (2^3)^6$

$= 2^{5x+10} \cdot 2^{18}$

$= 2^{5x+28}$

Ex. 3 Find t_n for each sequence.

This means find the general formula which works to find any term in the sequence. Must be simplified.

$t_n = ar^{n-1}$

a) 5, 10, 20, 40, ...

$a = 5$

$r = 2$

$t_n = ar^{n-1}$

$\boxed{t_n = 5 \cdot 2^{n-1}}$

b) 6561, 2187, 729, 243, ...

$a = 6561$

$r = \frac{2187}{6561}$

$= \frac{1}{3}$

$t_n = ar^{n-1}$

$= 6561 \cdot \left(\frac{1}{3}\right)^{n-1}$

$= 3^8 \left(\frac{1}{3}\right)^{n-1}$

$= \left(\frac{1}{3}\right)^{-8} \left(\frac{1}{3}\right)^{n-1}$ OR $= 3^8 (3)^{1-n}$

$\boxed{t_n = \left(\frac{1}{3}\right)^{n-9}}$ $\boxed{t_n = 3^{9-n}}$

c) 3, -12, 48, -192, ...

$a = 3$

$r = -4$

$t_n = ar^{n-1}$

$= 3 \cdot (-4)^{n-1}$

d) 8, 32, 128, 512,.....

$a = 8$

$r = 4$

$t_n = ar^{n-1}$

$= 8(4)^{n-1}$

$= 2^3 (2^2)^{n-1}$

$= 2^3 \, 2^{2n-2}$

$\boxed{t_n = 2^{2n+1}}$

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Ex. 4 Determine the number of terms in each sequence.

a) 5, 20, 80, ... , 81920

Same process as for arithmetic!
- Build t_n
- Solve for n of last term

$a = 5$ $\quad t_n = 5 \cdot 4^{n-1}$

$r = 4$

$81920 = 5 \cdot 4^{n-1}$

$16384 = 4^{n-1}$

$4^7 = 4^{n-1}$

$\therefore\ 7 = n - 1$

$8 = n$

$\therefore$ There are 8 terms

b) -19683, 6561, -2187, ... , -3

$a = -19683$

$r = \frac{t_2}{t_1}$

$= \frac{6561}{-19683}$

$= -\frac{1}{3}$

$t_n = ar^{n-1}$

$t_n = (-19683)\left(-\frac{1}{3}\right)^{n-1}$

$-3 = (-19683)\left(-\frac{1}{3}\right)^{n-1}$

$\frac{3}{19683} = \left(-\frac{1}{3}\right)^{n-1}$

$\frac{1}{6561} = \left(-\frac{1}{3}\right)^{n-1}$

$\left(-\frac{1}{3}\right)^8 = \left(-\frac{1}{3}\right)^{n-1}$

$\therefore\ 8 = n - 1$

$9 = n$

$\therefore$ There are 9 terms

Ex. 5 Determine a, r, and t_n for the geometric sequence that has:

a) $\underbrace{t_5 = 324 \quad \text{and} \quad t_9 = 26244}_{4 \text{ ratios}}$

324 ___ ___ ___ 26244

$\times r \quad \times r \quad \times r \quad \times r$

r^4

$$r^4 = \frac{26244}{324}$$
$$= 81$$
$$r = \pm\sqrt[4]{81}$$
$$\boxed{r = \pm 3}$$

Use t_5 to solve for a

$$t_n = ar^{n-1}$$
$$324 = a(\pm 3)^{5-1}$$
$$324 = a(\pm 3)^4$$
$$324 = 81a$$
$$\boxed{a = 4}$$

$$\therefore t_n = 4 \cdot (3)^{n-1}$$
OR
$$t_n = 4(-3)^{n-1}$$

b) $\underbrace{t_4 = -8 \quad \text{and} \quad t_7 = 1}_{3 \text{ ratios}}$

$$r^3 = \frac{1}{-8}$$
$$\boxed{r = -\frac{1}{2}}$$

Use t_7

$$t_n = ar^{n-1}$$
$$1 = a\left(-\tfrac{1}{2}\right)^{7-1}$$
$$1 = a\left(-\tfrac{1}{2}\right)^6$$
$$1 = a\left(\tfrac{1}{64}\right)$$
$$\boxed{64 = a}$$

$$t_n = ar^{n-1}$$
$$= 64\left(-\tfrac{1}{2}\right)^{n-1}$$
$$= (-2)^6(-2)^{1-n}$$
$$\boxed{t_n = (-2)^{7-n}}$$

Ex. 6 Determine the value of x that makes each sequence:

a) geometric

2, 6, 5x - 2

$$\frac{6}{2} = \frac{5x-2}{6}$$

$$18 = 5x - 2$$

$$4 = x$$

b) arithmetic

x - 4, 6, x

$$6-(x-4) = x-6$$

$$6-x+4 = x-6$$

$$16 = 2x$$

$$8 = x$$

Be careful of the wording in application problems:

Presently/Now -> t_1

First year -> t_2

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HOMEWORK
p. 430 # 1, 6i, 7, 11, 20