

## 6.4 - Arithmetic Series

Arithmetic Sequence: 2, 5, 8, 11, 14, ...

Arithmetic Series:  $2 + 5 + 8 + 11 + 14 + \dots$

An arithmetic series is the **SUM** of the terms in an arithmetic sequence.

$S_n$  represents the **sum** of the first **n** terms

eg. For  $2 + 5 + 8 + 11 + \dots$

$$\begin{array}{l} S_2 = t_1 + t_2 \\ = 2 + 5 \\ = 7 \end{array} \qquad \begin{array}{l} S_3 = t_1 + t_2 + t_3 \\ = 2 + 5 + 8 \\ = 15 \end{array}$$

### Development of the Arithmetic Series Formula:

Gauss added the numbers from 1 to 100 by:

$$\begin{array}{r} 1 + 2 + 3 + \dots + 99 + 100 \\ 100 + 99 + 98 + \dots + 2 + 1 \end{array}$$

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$$101 + 101 + 101 + \dots + 101 + 101$$

Each sum would = 101

There would be 100 sums all together... but this is two of the series added together so the sum is twice what it should be.

$$\begin{aligned} S_{100} &= \frac{100(101)}{2} \\ &= 5050 \end{aligned}$$

Try it out! Nice party trick :)



Johann Carl Friedrich Gauss  
German mathematician

In general, an arithmetic sequence is:  $a, a + d, a + 2d, + \dots + , t_n - d, t_n$

So, in general, the sum of an arithmetic sequence is:

$$S_n = a + (a + d) + (a + 2d) + \dots + (t_n - d) + t_n$$

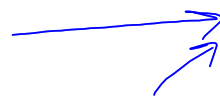
$$S_n = \frac{t_n + t_n - d + t_n - 2d + \dots + a + d + a}{2}$$

$$2S_n = (a + t_n) + (a + t_n) + (a + t_n) + \dots + (a + t_n) + (a + t_n)$$

$$a + d + t_n - d$$

$$2S_n = n(a + t_n)$$

$$S_n = \frac{n}{2}(a + t_n)$$



$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$t_n = a + (n-1)d$$

### Arithmetic Series Formulas

Any term in an arithmetic sequence/series,  $t_n$  can be found using:

$$t_n = a + (n - 1)d$$

Any sum in an arithmetic series,  $S_n$  can be found using:

$$S_n = \frac{n(a + t_n)}{2}$$

or

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

When we know  $t_n$

When we know  $d$

Ex. 1 Find the indicated sum for each series.

a)  $4 + 6 + 8 + 10 + \dots S_{42}$

Givens

$$a = 4$$

$$d = 2$$

$$n = 42$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{42}{2}(2(4) + (42-1)(2))$$

$$= 21(8 + 82)$$

$$= 1890$$

$$\therefore S_{42} = 1890$$

$5 + (-3) + (-11) + (-19) + \dots$

b)  $5 - 3 - 11 - 19 - \dots, S_{17}$

$a = 5$

$d = -8$

$n = 17$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{17} = \frac{17}{2}(2(5) + (16)(-8))$$

$$= -1003$$

$$\therefore S_{17} = -1003$$

Ex. 2 Find the sum of the series.

a)  $2 + 5 + 8 + 11 + \dots + 254$

$a = 2$

$d = 3$

$n = ?$

$t_n = 254$

Need to find n

$$t_n = a + (n-1)d$$

$$254 = 2 + (n-1)(3)$$

$$252 = (n-1)(3)$$

$$84 = n-1$$

$$85 = n$$

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{85} = \frac{85}{2}(2 + 254)$$

$$= 10880$$

$$\therefore S_{85} = 10880$$

b)  $5 + 3 + 1 - 1 - \dots - 401$

$a = 5$

$d = -2$

$n = ?$

$t_n = -401$

Find n

$$t_n = a + (n-1)d$$

$$-401 = 5 + (n-1)(-2)$$

$$-406 = (n-1)(-2)$$

$$203 = n-1$$

$$204 = n$$

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{204} = \frac{204}{2}(5 - 401)$$

$$= -40392$$

$$\therefore S_{204} = -40392$$

Ex. 3 Find the sum of the first 42 terms of an arithmetic series with  $t_1 = 7$  and  $t_{42} = 212$ .

Givens  
 $a = 7$   
 $t_n = 212$   
 $n = 42$

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{42} = \frac{42}{2}(7 + 212)$$

$$= 4599$$

$$\therefore S_{42} = 4599$$

## Homework

**p. 452 # 1, 2, 3, 5ace,  
6ace, 7ace, 11**

