6.4 - Arithmetic Series

Arithmetic Sequence: 2, 5, 8, 11, 14, ...

Arithmetic Series: 2 + 5 + 8 + 11 + 14 + ...

An arithmetic series is the SUM of the terms in an arithmetic sequence.

S_n represents the sum of the first n terms

eg. For 2 + 5 + 8 + 11 + ...

$S_2 = t_1 + t_2$
$= 2 + 5$
$= 7$

$S_3 = t_1 + t_2 + t_3$
$= 2 + 5 + 8$
$= 15$

Johann Carl Friedrich Gauss
German mathematician

Development of the Arithmetic Series Formula:

Gauss added the numbers from 1 to 100 by:

1 + 2 + 3 + ... + 99 + 100
100 + 99 + 98 + ... + 2 + 1

101 + 101 + 101 + ... + 101 + 101

Each sum would = 101
There would be 100 sums all together... but this is two of the series added together so the sum is twice what it should be.

$$S_{100} = \frac{100(101)}{2}$$
$$= 5050$$

Try it out! Nice party trick :)

In general, an arithmetic sequence is: $a, a + d, a + 2d, + \ldots +, t_n - d, t_n$

So, in general, the sum of an arithmetic sequence is:

$S_n = a + (a + d) + (a + 2d) + \ldots + (t_n - d) + t_n$

$S_n = t_n + t_n - d + t_n - 2d + \ldots + a + d + a$

$2S_n = (a + t_n) + (a + t_n) + (a + t_n) + \ldots + (a + t_n) + (a + t_n)$

$a + d + t_n - d$

$2S_n = n(a + t_n)$

$\boxed{S_n = \frac{n}{2}(a + t_n)} \longrightarrow \boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$

$t_n = a + (n-1)d$

Arithmetic Series Formulas

Any term in an arithmetic sequence/series, t_n can be found using:

$$t_n = a + (n - 1)d$$

Any sum in an arithmetic series, S_n can be found using:

$$S_n = \frac{n(a + t_n)}{2} \quad \text{or} \quad S_n = \frac{n}{2}[2a + (n-1)d]$$

When we know t_n When we know d

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Ex. 1 Find the indicated sum for each series.

a) 4 + 6 + 8 + 10 +... S_{42}

Givens

$a = 4$
$d = 2$
$n = 42$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$= \frac{42}{2}(2(4) + (42-1)(2))$$
$$= 21(8 + 82)$$
$$= 1890$$

$\therefore S_{42} = 1890$

$5 + (-3) + (-11) + (-19) + \ldots$

b) 5 - 3 - 11 - 19 - ..., S_{17}

$a = 5$
$d = -8$
$n = 17$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_{17} = \frac{17}{2}(2(5) + (16)(-8))$$
$$= -1003$$

$\therefore S_{17} = -1003$

Ex. 2 Find the sum of the series.

a) 2 + 5 + 8 + 11 + ... + 254

$a = 2$
$d = 3$
$n = ?$
$t_n = 254$

Need to find n

$$t_n = a + (n-1)d$$
$$254 = 2 + (n-1)(3)$$
$$252 = (n-1)(3)$$
$$84 = n - 1$$
$$85 = n$$

$$S_n = \frac{n}{2}(a + t_n)$$
$$S_{85} = \frac{85}{2}(2 + 254)$$
$$= 10880$$

$\therefore S_{85} = 10880$

b) 5 + 3 + 1 - 1 - ... - 401

$a = 5$
$d = -2$
$n = ?$
$t_n = -401$

Find n

$$t_n = a + (n-1)d$$
$$-401 = 5 + (n-1)(-2)$$
$$-406 = (n-1)(-2)$$
$$203 = n - 1$$
$$204 = n$$

$$S_n = \frac{n}{2}(a + t_n)$$
$$S_{204} = \frac{204}{2}(5 - 401)$$
$$= -40392$$

$\therefore S_{204} = -40392$

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Ex. 3 Find the sum of the first 42 terms of an arithmetic series with $t_1 = 7$ and $t_{42} = 212$.

Givens

$a = 7$

$t_n = 212$

$n = 42$

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{42} = \frac{42}{2}(7 + 212)$$

$$= 4599$$

$$\therefore S_{42} = 4599$$

Homework

p. 452 # 1, 2, 3, 5ace, 6ace, 7ace, 11