6.5 - Geometric Series

4, 8, 16, 32, ... geometric sequence

4 + 8 + 16 + 32 + ... geometric series: The **sum** of the terms of a geometric sequence.

Derivation of the Geometric Series Formula

$$S_n = a + ar + ar^2 + ... + ar^{n-1}$$
$$rS_n = \quad ar + ar^2 + ... + ar^{n-1} + ar^n$$

$S_n - rS_n = a - ar^n$

$S_n(1-r) = a(1-r^n)$

$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Geometric Series Formulas

Any term, t_n, can be found using: $t_n = a\,r^{n-1}$

Any sum, S_n, can be found using:

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{OR} \quad \frac{a(1 - r^n)}{1 - r}, \text{ where } r \neq 1$$

and:
- a = first term
- r = common ratio
- n = # of terms

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Ex. 1 Determine the indicated sum of each series.

$4 + (-8) + 16 + (-\ldots$

a) $4 - 8 + 16 - \ldots$, S_9

$a = 4$

$r = \frac{-8}{4}$

$= -2$

$n = 9$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_9 = \frac{4(1-(-2)^9)}{1-(-2)}$$

$$= \frac{4(1-(-512))}{3}$$

$$= \frac{4(513)}{3}$$

$$= 684$$

$\therefore S_9 = 684$

b) $64 + 32 + 16 + \ldots$, S_{12}

$a = 64$

$n = 12$

$r = \frac{32}{64}$

$= \frac{1}{2}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{12} = \frac{64\left(1-\left(\frac{1}{2}\right)^{12}\right)}{1-\frac{1}{2}}$$

$$= \frac{64\left(1-\frac{1}{4096}\right)}{\frac{1}{2}}$$

$$= {}^{1}\cancel{64}\left(\frac{4095}{\cancel{4096}_{64}}\right) \div \frac{1}{2}$$

$$= \frac{4095}{\cancel{64}_{32}} \times \frac{\cancel{2}^{1}}{1}$$

$$= \frac{4095}{32}$$

$\therefore S_{12} = \frac{4095}{32}$

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Ex. 2 Determine the sum of the series.

$2 + 6 + 18 + \ldots + 4374$

$a = 2$

$r = 3$

$n = ?$

Find n $\left(t_n = ar^{n-1}\right)$

$4374 = 2(3)^{n-1}$

$2187 = 3^{n-1}$

$3^7 = 3^{n-1}$

$\therefore 7 = n-1$

$8 = n$

Find S_8

$S_n = \frac{a(1-r^n)}{1-r}$

$S_8 = \frac{2(1-3^8)}{1-3}$

$= \frac{2(-6560)}{-2}$

$= 6560$

$\therefore S_8 = 6560$

Ex. 3 What if r = 1? Suppose a = 5 and r =1, find S_{10} for the series.

$5 + 5 + 5 + 5 + \ldots\ldots$

$S_{10} = 5(10)$

$= 50$

Can't use

$S_n = \frac{a(1-r^n)}{1-r}$

Divide by zero

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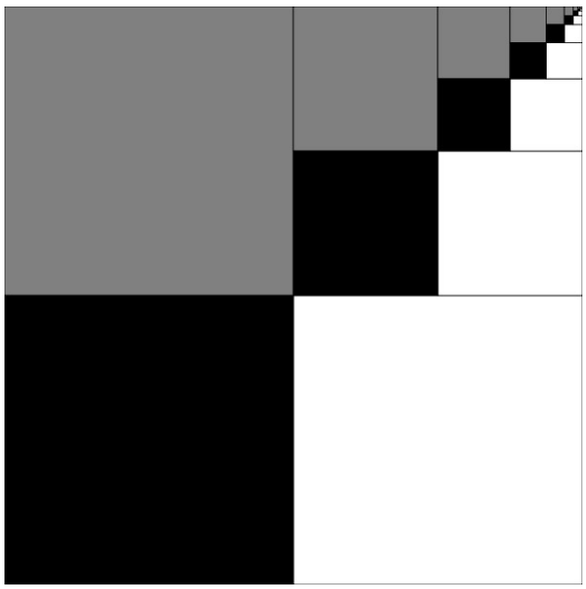
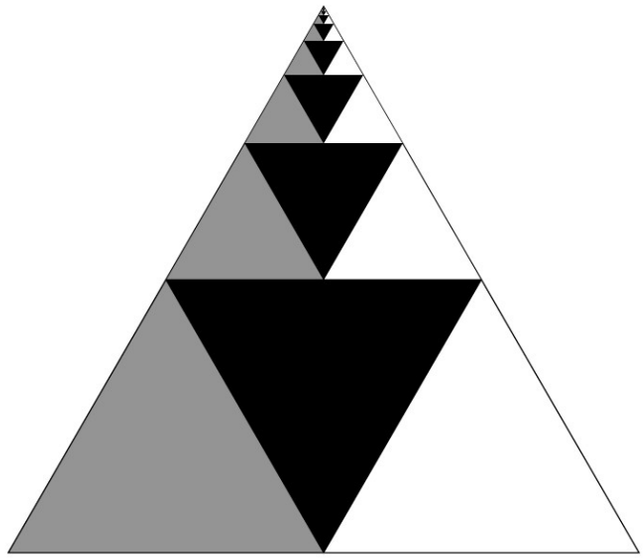
Homework
p. 459 # 4, 5abe, 7, 9, 11, 13 *challenge





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