6.7 - The Binomial Theorem

You have actually used this theorem before...

$(a + b)^2$

$= 1a^2 + 2ab + 1b^2$

$(a + b)^3$

$= (a+b)^2(a+b)$

$= (a^2+2ab+b^2)(a+b)$

$= a^3 + a^2b + 2a^2b + 2ab^2 + ab^2 + b^3$

$= 1a^3 + 3a^2b + 3ab^2 + 1b^3$

Investigate the expansion of:

$(a + b)^4$

$= (a+b)^3(a+b)$

$= (a^3+3a^2b+3ab^2+b^3)(a+b)$

$= a^4+a^3b + 3a^3b + 3a^2b^2 + 3a^2b^2 + 3ab^3 + ab^3 + b^4$

$= 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$

OR $(a+b)^2(a+b)^2$

$= (a^2+2ab+b^2)(a^2+2ab+b^2)$

	a^2	$2ab$	b^2
a^2	a^4	$2a^3b$	a^2b^2
$2ab$	$2a^3b$	$4a^2b^2$	$2ab^3$
b^2	a^2b^2	$2ab^3$	b^4

<u>In general</u>:

n value (exponent)	$(a + b)^n$	Coefficients
0	$(a+b)^0 = 1$	1
1	$(a+b)^1 = 1a + 1b$	1 1
2	$(a+b)^2 =$	1 2 1
3	$(a+b)^3 =$	1 3 3 1
	$\vdots$	$\vdots$

The Binomial Theorem

The binomial theorem is used to expand $(a + b)^n$.

The coefficients are the values of row n in Pascal's Triangle (where *n* is the exponent to which the binomial is being raised).

$(a + b)^n$

$$= \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}a^0 b^n$$

Ex. 1 Expand each of the following :

a) $(a + b)^6$

$$= \binom{6}{0}a^6b^0 + \binom{6}{1}a^5b^1 + \binom{6}{2}a^4b^2 + \binom{6}{3}a^3b^3 + \binom{6}{4}a^2b^4$$
$$+ \binom{6}{5}a^1b^5 + \binom{6}{6}a^0b^6$$

$$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

b) $(x - 2)^4$

$$= \binom{4}{0}x^4(-2)^0 + \binom{4}{1}x^3(-2)^1 + \binom{4}{2}x^2(-2)^2 + \binom{4}{3}x^1(-2)^3 + \binom{4}{4}x^0(-2)^4$$

$$= x^4 + (4)x^3(-2) + (6)x^2(4) + (4)x(-8) + 16$$

$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$

c) The first 3 terms of

$(3x+2y)^5$

$a = 3x$
$b = 2y$

$= \binom{5}{0}(3x)^5(2y)^0 + \binom{5}{1}(3x)^4(2y)^1 + \binom{5}{2}(3x)^3(2y)^2 + \ldots$

$= (1)(243x^5)(1) + (5)(81x^4)(2y) + (10)(27x^3)(4y^2) + \ldots$

$= 243x^5 + 810x^4y + 1080x^3y^2 + \ldots$

d) $\left(x^2 + \frac{4}{x}\right)^3$

$a = x^2$
$b = 4x^{-1}$

e) The first four terms of

$$\left(\sqrt{x}-\frac{2}{\sqrt{x^3}}\right)^8$$

$a = x^{\frac{1}{2}}$

$b = -2x^{-\frac{3}{2}}$

$= \binom{8}{0}\left(x^{\frac{1}{2}}\right)^8\left(-2x^{-\frac{3}{2}}\right)^0 + \binom{8}{1}\left(x^{\frac{1}{2}}\right)^7\left(-2x^{-\frac{3}{2}}\right)^1 + \ldots\ldots$

Ex. 2 Write $1 + 12x^3 + 54x^6 + 108x^9 + 81x^{12}$ in the form $(a+b)^n$

May 20-11:01 PM

HOMEWORK
p. 344 #C4, 5acef, 6, 7, 19

Dec 13-10:24 PM