

2.3 Solving Systems by Substitution

Remember! "Solving a system" means finding a coordinate they both share. A single (x,y).

What's so great about solving algebraically?



- gives exact values
- less time / less space

Ex. 1 How would you solve

① $x = 5$

② $3x - 4y = 3$



Sub ① into ②

$$3(5) - 4y = 3$$

$$15 - 4y = 3$$

$$-4y = 3 - 15$$

$$\frac{-4y}{-4} = \frac{-12}{-4}$$

$$y = 3$$

$\therefore (5, 3)$

THE SUBSTITUTION METHOD:

1. **Isolate** a variable in one equation (pick the best one)
2. **Substitute** to create an equation with only one variable.
3. **Solve** the equation.
4. **Substitute** the solved variable into the equation from #1 to determine the value of the other variable.
5. **Write** a conclusion.
6. **(Check)** - formal if asked, otherwise complete a mental check.

Ex. 3 Solve using the substitution method.

$$\begin{aligned} \text{a) } x + 3y &= -4 & \textcircled{1} \\ 2x - 3y &= 1 & \textcircled{2} \end{aligned}$$

$$\begin{aligned} \textcircled{1} \quad x + 3y &= -4 \\ x &= -4 - 3y \end{aligned}$$

$$\begin{aligned} \text{Sub } x &= -4 - 3y \text{ into } \textcircled{2} \\ 2(-4 - 3y) - 3y &= 1 \\ -8 - 6y - 3y &= 1 \\ -8 - 9y &= 1 \\ -9y &= 9 \\ y &= -1 \end{aligned}$$

$$\begin{aligned} \text{Sub } y &= -1 \text{ into } \textcircled{1} \\ x &= -4 - 3(-1) \\ &= -4 + 3 \\ &= -1 \end{aligned}$$

$$\therefore (-1, -1)$$

$$\begin{aligned} \text{b) } 5a + 3b &= 10 & \textcircled{1} \\ 2a - b &= 4 & \textcircled{2} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 2a - b &= 4 \\ 2a &= 4 + b \\ 2a - 4 &= b \end{aligned}$$

$$\begin{aligned} \text{Sub } \textcircled{2} \text{ into } \textcircled{1} \\ 5a + 3(2a - 4) &= 10 \\ 5a + 6a - 12 &= 10 \\ 11a - 12 &= 10 \\ 11a &= 22 \\ a &= 2 \end{aligned}$$

$$\begin{aligned} \text{Sub } a &= 2 \text{ into } \textcircled{2} \\ b &= 2(2) - 4 \\ &= 0 \end{aligned}$$

$$\therefore \begin{aligned} a &= 2 \\ b &= 0 \end{aligned}$$

SPECIAL CASES

A. $x + y = 8$ ①
 $3x + 3y = -5$ ②

① $x + y = 8$
 $y = 8 - x$

Sub into ②

$$3x + 3(8 - x) = -5$$

$$3x + 24 - 3x = -5$$

$$3x - 3x = -5 - 24$$

$$0x = -29 \quad ????$$

$\therefore$ NO SOLUTION!

B. $p - 2q = -3$ ①
 $4q = 2p + 6$ ②

① $p - 2q = -3$
 $p = -3 + 2q$

Sub into ②

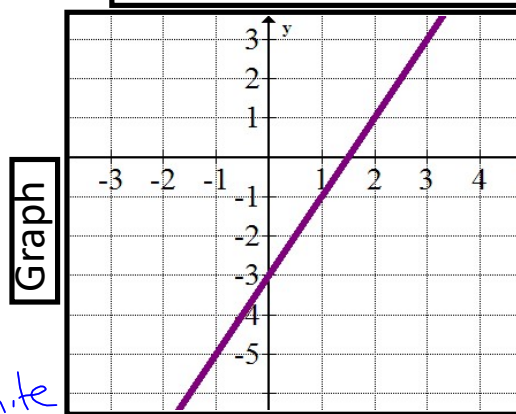
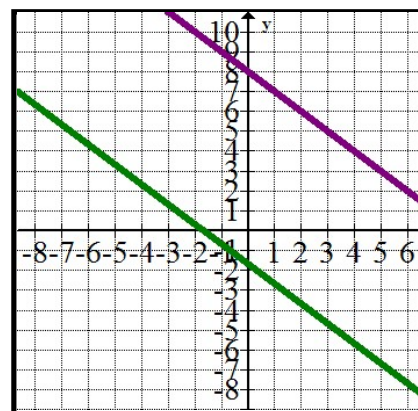
$$4q = 2(-3 + 2q) + 6$$

$$4q = -6 + 4q + 6$$

$$4q - 4q = -6 + 6$$

$$0q = 0$$

$\therefore$ Infinite
 Solutions!
 (same line)



Working with fractions...
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$$x + 2y = 3 \quad (1)$$

$$5x + 4y = 8 \quad (2)$$

$$\begin{aligned} (1) \quad x + 2y &= 3 \\ x &= 3 - 2y \end{aligned}$$

Sub into (2)

$$\begin{aligned} 5(3 - 2y) + 4y &= 8 \\ 15 - 10y + 4y &= 8 \\ -6y &= -7 \\ y &= \frac{7}{6} \end{aligned}$$

Challenge!

$$\text{ans: } \left(\frac{2}{3}, \frac{7}{6} \right)$$

Sub into (1)

$$\begin{aligned} x &= 3 - 2\left(\frac{7}{6}\right) \\ &= 3 - \frac{14}{6} \\ &= \frac{18}{6} - \frac{14}{6} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

Homework:

Set 1: p.26 #1ab,2d,4bd,5ac,12

Set 2: p.26 #5de,12,17a,19,20b