

2.4 Types of Intersections & Equivalent Systems

A: Types of Intersections

How many solutions can a system of linear equations have?

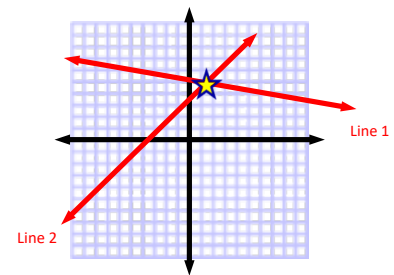
How can you tell just by looking at the equations? Explain your answer.

A linear system can have:

ONE SOLUTION The lines intersect at one point

The two linear equations have:

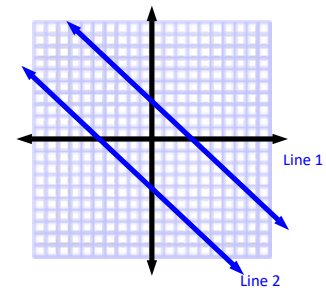
- different slopes
- y-intercept does not matter



NO SOLUTION The lines do not intersect

The two linear equations have:

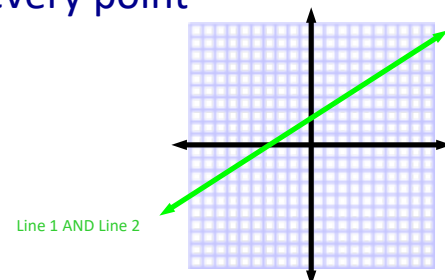
- same slopes
- different y-intercepts



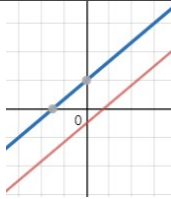
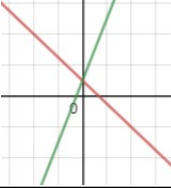
INFINITE SOLUTIONS The lines intersect at every point

The two linear equations have:

- same slopes
- same y-intercepts



Ex. 1 Complete the table.

System #	Equations	Slope	y-int	# of intersections	Solution
1	$y = \frac{2}{3}x - 1$	$\frac{2}{3}$	-1	0	
	$y = \frac{2}{3}x + 2$	$\frac{2}{3}$	2		
2	$y = -\frac{1}{2}x - 1$	$-\frac{1}{2}$	-1	∞	
	$x + 2y = -2$	$-\frac{1}{2}$	-1		
3	$y = -\frac{3}{4}x + 1$	$-\frac{3}{4}$		1	
	$y = 2x + 1$	2			

What kind of equations give an INFINITE number of solutions?

-same slope and same y-intercept

2b) $x + 2y = -2$
 $2y = -x - 2$
 $y = -\frac{1}{2}x - 1$

Ex. 2 For what value of p will the system of linear equations have **NO solution**?

a) $x - 2y = 4$ ①
 $y = px + 1$ ②

- Parallel (Same slope)

- Different y-ints ✓

① $x - 2y = 4$
 $-2y = -x + 4$
 $y = \frac{1}{2}x - 2$

OR
 $x - 4 = 2y$
 $\frac{1}{2}x - 2 = y$
 $y = \frac{1}{2}x - 2$
 $m = \frac{1}{2}$
 $\therefore p = \frac{1}{2}$

b) $3x + y = 1$ ①
 $x + y = p(x + 2)$ ②

- Same slope

- Different y-ints

① $3x + y = 1$
 $y = -3x + 1$
 slope = -3
 y-int = 1

② $x + y = p(x + 2)$
 $x + y = px + 2p$
 $y = px - x + 2p$
 $= x(p - 1) + 2p$
 $= \underbrace{(p - 1)}_m x + \underbrace{2p}_b$

In order to have parallel lines, need same slope!

$-3 = p - 1$
 $-2 = p$

confirm b is different than 1
 if $p = -2$

$b = 2p$
 $= 2(-2)$
 $= -4$

B: Equivalent Systems...

Yesterday, I needed 3 Timmy's coffees and 2 donuts to get the day started. That cost me \$8.75. Today, I only needed one coffee and two donuts which cost \$4.25. What is the cost of a coffee and a donut?



- Let c be the cost of a coffee
 — Let d " " " " " donut

$$\textcircled{1} \quad 3c + 2d = 8.75$$

$$\textcircled{2} \quad c + 2d = 4.25$$

$$\textcircled{2} \quad c + 2d = 4.25$$

$$c = 4.25 - 2d$$

Sub into $\textcircled{1}$

$$3(4.25 - 2d) + 2d = 8.75$$

$$12.75 - 6d + 2d = 8.75$$

$$-4d = 8.75 - 12.75$$

$$-4d = -4$$

$$d = 1$$

Sub $d = 1$ into $\textcircled{2}$

$$c = 4.25 - 2(1) \\ = 2.25$$

$\therefore$ A coffee is \$2.25

A donut is \$1

Two Tim's lattes and two cookies cost \$7.10. Five lattes and four cookies cost \$16.95. Determine the cost of a latte and a cookie.

Let l be the cost of latte

Let c be the cost of cookie

$$2l + 2c = 7.10$$

$$5l + 4c = 16.95$$

Answers

$$l = 2.75$$

$$c = 0.80$$

Homework:

Set 1: p. 32 #1-3, 6

Set 2: p. 32 #2, 3d, 8