

2.5 Solving by Elimination

What were the big ideas from yesterday's lesson and homework?



- An equivalent linear system is formed by multiplying/dividing any of the equations by a constant.



- An equivalent linear system is formed by adding or subtracting the equations.

➔ Eliminate means to remove or get rid of.
What do you think we would like to eliminate?

Consider the following system:

$$\begin{array}{r} x - y = 1 \\ + \quad 3x + y = 11 \\ \hline 4x + 0y = 12 \\ 4x = 12 \\ x = 3 \end{array}$$



To add or to subtract?
Think...

What happens when we add the equations?

We eliminated the y-term
Why?

We deliberately set it up to
cancel the y-term

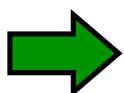
$$\begin{array}{r} \boxed{-} \quad y \\ \quad y \\ \hline 0y \end{array}$$

$$\begin{array}{r} \boxed{+} \quad -y \\ \quad y \\ \hline 0y \end{array}$$

$$\begin{array}{r} \boxed{+} \quad y \\ \quad -y \\ \hline 0y \end{array}$$

$$\begin{array}{r} \boxed{-} \quad -y \\ \quad -y \\ \hline 0y \end{array}$$

$$\begin{aligned} & -y - (-y) \\ & = -y + y \\ & = 0 \end{aligned}$$



Opposite signs --> add
Same signs --> subtract



Examples: Solve by elimination.

$$\begin{array}{r} \text{a. } x - y = 1 \\ + \quad 3x + y = 11 \\ \hline 4x + 0 = 12 \\ 4x = 12 \\ x = 3 \end{array}$$

→ Sub $x=3$ into either eqⁿ
(So pick the easiest!)

$$\begin{array}{r} x - y = 1 \\ 3 - y = 1 \\ -y = -2 \\ y = 2 \end{array}$$

∴ (3, 2)

METHOD 2 - THE ELIMINATION METHOD

1. Multiply one or both equations by a constant so that the coefficients of either x or y are the same in both equations (sign does not matter).
2. Add or subtract the equations to *eliminate* one variable.
3. Solve the remaining equation.
4. Substitute the solved value into one of the original equations to determine the value of the other variable.
5. Write a conclusion.
6. Check (formally if asked, otherwise mentally).

$$\begin{array}{l} \text{b. } x + 3y = 2 \quad (1) \\ 2x + 5y = 3 \quad (2) \end{array}$$

$$\begin{array}{r} (1) \times 2 \quad 2x + 6y = 4 \\ (2) \quad -2x + 5y = 3 \\ \hline \quad \quad \quad 0 + y = 1 \\ \quad \quad \quad y = 1 \end{array}$$

$$\begin{array}{l} \text{Sub } y=1 \text{ into } (1) \\ x + 3(1) = 2 \\ x + 3 = 2 \\ x = -1 \end{array}$$

$$\therefore (-1, 1)$$

$$\begin{array}{l} \text{c. } 5x - 3y = 9 \quad (1) \\ 2x - 5y = -4 \quad (2) \end{array}$$

$$\begin{array}{r} (1) \times 2 \quad 10x - 6y = 18 \\ (2) \times 5 \quad -10x - 25y = -20 \\ \hline \quad \quad \quad 19y = 38 \\ \quad \quad \quad y = 2 \end{array}$$

Sub into (2)

$$\begin{array}{l} 2x - 5(2) = -4 \\ 2x - 10 = -4 \\ \quad \quad \quad +10 \quad \quad +10 \\ 2x = 6 \\ x = 3 \end{array}$$

$$\therefore (3, 2)$$

$$d. \begin{cases} 2x + 3y = 8 & (1) \\ 3x - 5y = 2 & (2) \end{cases}$$

$$3x - 5y = 2 \quad (2)$$

$$(1) \times 3 \quad 6x + 9y = 24$$

$$(2) \times 2 \quad \underline{6x - 10y = 4}$$

$$19y = 20$$

$$y = \frac{20}{19}$$

Sub into (1)

$$2x + 3\left(\frac{20}{19}\right) = 8$$

$$2x + \frac{60}{19} = 8$$

$$2x = 8 - \frac{60}{19}$$

$$2x = \frac{152}{19} - \frac{60}{19}$$

$$2x = \frac{92}{19}$$

$$x = \frac{1}{2}\left(\frac{92}{19}\right)$$

$$= \frac{92 \div 2}{38 \div 2}$$

$$\therefore \left(\frac{46}{19}, \frac{20}{19}\right)$$

$$= \frac{46}{19}$$

$$e. \begin{cases} 3m = -1 - 4n \\ 5n = 4m + 22 \end{cases}$$

$$5n = 4m + 22$$

Rearrange First!

$$(1) \quad 3m + 4n = -1$$

$$(2) \quad -4m + 5n = 22$$

$$(1) \times 4 \quad 12m + 16n = -4$$

$$(2) \times 3 \quad \underline{+ -12m + 15n = 66}$$

$$0 + 31n = 62$$

$$n = 2$$

Sub $n=2$ into (1)

$$3m + 4(2) = -1$$

$$3m + 8 = -1$$

$$3m = -9$$

$$m = -3$$

$$\therefore \begin{cases} m = -3 \\ n = 2 \end{cases}$$



The Eliminator

... solving systems by
eliminating one variable at
a time!

Homework:

Set 1: p.40 #2d, 4a, 5ab, 6ad, 7c

Set 2: p.40 #5d, 6b, 7c, 12c, 13, 18, 19b, 20a