Answers

Chapter 1

Getting Started, p. 2

1. **a)** 6 **c)** $-\frac{51}{16}$

b) -6 **d)** $a^2 + 5a$

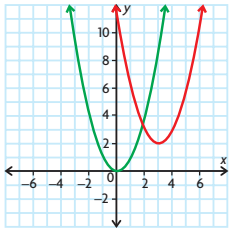
2. **a)** $(x + y)(x + y)$

b) $(5x - 1)(x - 3)$

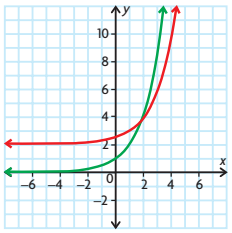
c) $(x + y + 8)(x + y - 8)$

d) $(a + b)(x - y)$

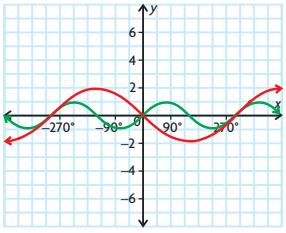
3. **a)** horizontal translation 3 units to the right, vertical translation 2 units up;



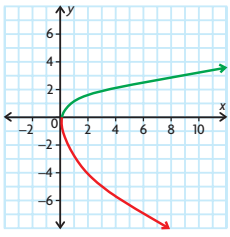
b) horizontal translation 1 unit to the right, vertical translation 2 units up;



c) horizontal stretch by a factor of 2, vertical stretch by a factor of 2, reflection across the x-axis;



d) horizontal compression by a factor of $\frac{1}{2}$, vertical stretch by a factor of 2, reflection across the x-axis;



4. **a)** $D = \{x \in \mathbf{R} \mid -2 \le x \le 2\}$, $R = \{y \in \mathbf{R} \mid 0 \le y \le 2\}$

b) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y \ge -19\}$

c) $D = \{x \in \mathbf{R} \mid x \ne 0\}$, $R = \{y \in \mathbf{R} \mid y \ne 0\}$

d) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid -3 \le y \ge 3\}$

e) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y > 0\}$

5. **a)** This is not a function; it does not pass the vertical line test.

b) This is a function; for each x-value, there is exactly one corresponding y-value.

c) This is not a function; for each x-value greater than 0, there are two corresponding y-values.

d) This is a function; for each x-value, there is exactly one corresponding y-value.

e) This is a function; for each x-value, there is exactly one corresponding y-value.

6. **a)** 8

b) about 2.71

7. If a relation is represented by a set of ordered pairs, a table, or an arrow diagram, one can determine if the relation is a function by checking that each value of the independent variable is paired with no more than one value of the dependent variable. If a relation is represented using a graph or scatter plot, the vertical line test can be used to determine if the relation is a function. A relation may also be represented by a description/rule or by using function notation or an equation. In these cases, one can use reasoning to determine if there is more than one value of the dependent variable paired with any value of the independent variable.

Lesson 1.1, pp. 11–13

1. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid -4 \le y \le -2\}$; This is a function because it passes the vertical line test.

b) $D = \{x \in \mathbf{R} \mid -1 \le x \le 7\}$; $R = \{y \in \mathbf{R} \mid -3 \le y \le 1\}$; This is a function because it passes the vertical line test.

c) $D = \{1, 2, 3, 4\}$; $R = \{-5, 4, 7, 9, 11\}$; This is not a function because 1 is sent to more than one element in the range.

d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$; This is a function because every element in the domain produces exactly one element in the range.

e) $D = \{-4, -3, 1, 2\}$; $R = \{0, 1, 2, 3\}$; This is a function because every element of the domain is sent to exactly one element in the range.

f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \le 0\}$; This is a function because every element in the domain produces exactly one element in the range.

2. **a)** $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \le -3\}$; This is a function because every element in the domain produces exactly one element in the range.

b) $D = \{x \in \mathbf{R} \mid x \ne -3\}$; $R = \{y \in \mathbf{R} \mid y \ne 0\}$; This is a function because every element in the domain produces exactly one element in the range.

c) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y > 0\}$; This is a function because every element in the domain produces exactly one element in the range.

d) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid 0 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.

e) $D = \{x \in \mathbf{R} \mid -3 \le x \le 3\}$; $R = \{y \in \mathbf{R} \mid -3 \le y \le 3\}$; This is not a function because (0, 3) and (0, 3) are both in the relation.

f) $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid -2 \le y \le 2\}$; This is a function because every element in the domain produces exactly one element in the range.

3. **a)** function; $D = \{1, 3, 5, 7\}$; $R = \{2, 4, 6\}$

b) function; $D = \{0, 1, 2, 5\}$; $R = \{-1, 3, 6\}$

c) function; $D = \{0, 1, 2, 3\}$; $R = \{2, 4\}$

d) not a function; $D = \{2, 6, 8\}$; $R = \{1, 3, 5, 7\}$

e) not a function; $D = \{1, 10, 100\}$; $R = \{0, 1, 2, 3\}$

f) function; $D = \{1, 2, 3, 4\}$; $R = \{1, 2, 3, 4\}$

4. **a)** function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \ge 2\}$.

b) not a function; $D = \{x \in \mathbf{R} \mid x \ge 2\}$; $R = \{y \in \mathbf{R}\}$

c) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R} \mid y \ge -0.5\}$

d) not a function; $D = \{x \in \mathbf{R} \mid x \ge 0\}$; $R = \{y \in \mathbf{R}\}$

e) function; $D = \{x \in \mathbf{R} \mid x \ne 0\}$; $R = \{y \in \mathbf{R} \mid y \ne 0\}$

f) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

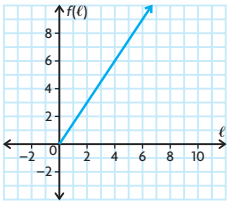
5. **a)** $y = x + 3$ **c)** $y = 3(x - 2)$

b) $y = 2x - 5$ **d)** $y = -x + 5$

6. **a)** The length is twice the width.

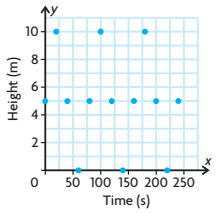
b) $f(l) = \frac{3}{2}l$

c)



d) length = 8 m; width = 4 m

7. **a)**

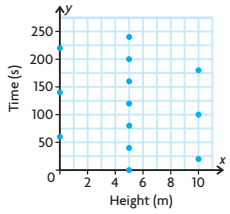


b) D = {0, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240}

c) R = {0, 5, 10}

d) It is a function because it passes the vertical line test.

e)



f) It is not a function because (5, 0) and (5, 40) are both in the relation.

8. **a)** {(1, 2), (3, 4), (5, 6)}

b) {(1, 2), (3, 2), (5, 6)}

c) {(2, 1), (2, 3), (5, 6)}

9. If a vertical line passes through a function and hits two points, those two points have identical *x*-coordinates and different *y*-coordinates. This means that one *x*-coordinate is sent to two different elements in the range, violating the definition of *function*.

10. **a)** Yes, because the distance from (4, 3) to (0, 0) is 5.

b) No, because the distance from (1, 5) to (0, 0) is not 5.

c) No, because (4, 3) and (4, −3) are both in the relation.

11. **a)** $g(x) = x^2 + 3$

b) $g(3) - g(2) = 12 - 7$
$= 5$
$g(3 - 2) = g(1)$
$= 4$
So, $g(3) - g(2) \neq g(3 - 2)$.

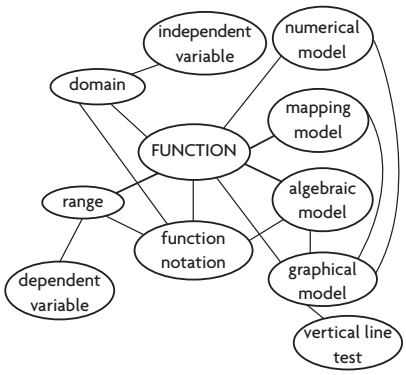
12. **a)** $f(6) = 12; f(7) = 8; f(8) = 15$

b) Yes, $f(15) = f(3) \times f(5)$

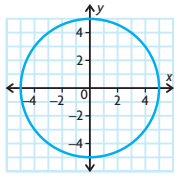
c) Yes, $f(12) = f(3) \times f(4)$

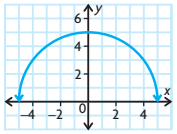
d) Yes, there are others that will work. $f(a) \times f(b) = f(a \times b)$ whenever a and b have no common factors other than 1.

13. Answers may vary. For example:



14.





The first is not a function because it fails the vertical line test:
D = $\{x \in \mathbf{R} \mid -5 \leq x \leq 5\}$;
R = $\{y \in \mathbf{R} \mid -5 \leq y \leq 5\}$.
The second is a function because it passes the vertical line test:
D = $\{x \in \mathbf{R} \mid -5 \leq x \leq 5\}$;
R = $\{y \in \mathbf{R} \mid 0 \leq y \leq 5\}$.

15. *x* is a function of *y* if the graph passes the horizontal line test. This occurs when any horizontal line hits the graph at most once.

Lesson 1.2, p. 16

1. $|-5|, |12|, |-15|, |20|, |-25|$

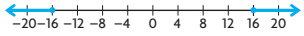
2. **a)** 22 **c)** 18 **e)** −2
b) −35 **d)** 11 **f)** −2

3. **a)** $|x| > 3$ **c)** $|x| \geq 1$
b) $|x| \leq 8$ **d)** $|x| \neq 5$

4. **a)**

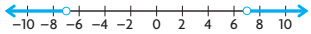


b)



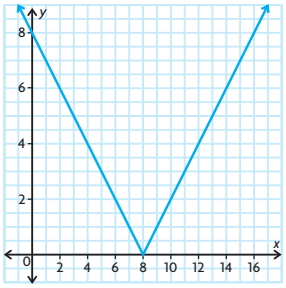
c) The absolute value of a number is always greater than or equal to 0. There are no solutions to this inequality.

d)



5. **a)** $|x| \leq 3$ **c)** $|x| \geq 2$
b) $|x| > 2$ **d)** $|x| < 4$

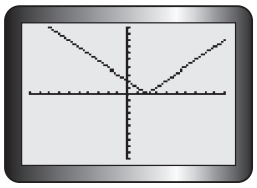
6.



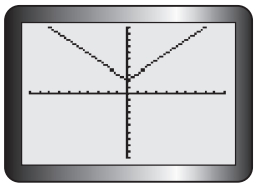
a) The graphs are the same.

b) Answers may vary. For example, $x - 8 = -(-x + 8)$, so they are negatives of each other and have the same absolute value.

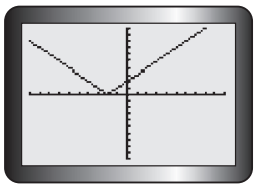
7. **a)**



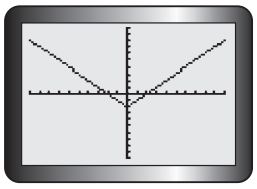
b)



c)



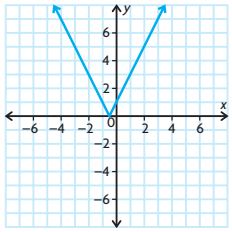
d)



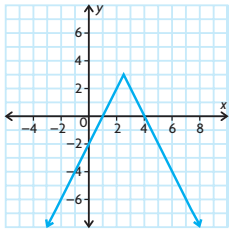
8. When the number you are adding or subtracting is inside the absolute value signs, it moves the function to the left (when adding) or to the right (when subtracting) of the origin. When the number you are

adding or subtracting is outside the absolute value signs, it moves the function down (when subtracting) or up (when adding) from the origin.
The graph of the function will be the absolute value function moved to the left 3 units and up 4 units from the origin.

9. This is the graph of $g(x) = |x|$ horizontally compressed by a factor of $\frac{1}{2}$ and translated $\frac{1}{2}$ unit to the left.



10. This is the graph of $g(x) = |x|$ horizontally compressed by a factor of $\frac{1}{2}$, reflected over the x-axis, translated $2\frac{1}{2}$ units to the right, and translated 3 units up.



Lesson 1.3, pp. 23–25

1. Answers may vary. For example, domain because most of the parent functions have all real numbers as a domain.

2. Answers may vary. For example, the end behaviour because the only two that match are x^2 and $|x|$.

3. Given the horizontal asymptote, the function must be derived from 2^x. But the asymptote is at $y = 2$, so it must have been translated up two. Therefore, the function is $f(x) = 2^x + 2$.

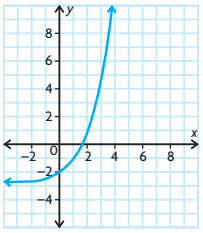
4. **a)** Both functions are odd, but their domains are different.
b) Both functions have a domain of all real numbers, but $\sin(x)$ has more zeros.
c) Both functions have a domain of all real numbers, but different end behaviour.
d) Both functions have a domain of all real numbers, but different end behaviour.

5. **a)** even **d)** odd
b) odd **e)** neither even nor odd
c) odd **f)** neither even nor odd

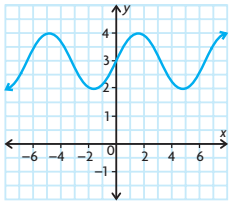
6. **a)** $|x|$, because it is a measure of distance from a number
b) $\sin(x)$, because the heights are periodic
c) 2^x, because population tends to increase exponentially
d) x, because there is \$1 on the first day, \$2 on the second, \$3 on the third, etc.

7. **a)** $f(x) = \sqrt{x}$ **c)** $f(x) = x^2$
b) $f(x) = \sin x$ **d)** $f(x) = x$

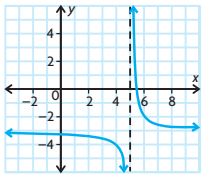
8. **a)** $f(x) = 2^x - 3$



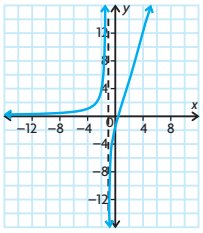
b) $g(x) = \sin x + 3$



c) $h(x) = \dfrac{1}{x-5} - 3 = \dfrac{16-3x}{x-5}$



9.



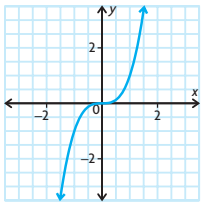
10. **a)** $f(x) = (x-2)^2$
b) There is not only one function.
$f(x) = \frac{3}{4}(x-2)^2 + 1$ works as well.
c) There is more than one function that satisfies the property.
$f(x) = |x-2| + 2$ and
$f(x) = 2|x-2|$ both work.

11. x^2 is a smooth curve, while $|x|$ has a sharp, pointed corner at (0, 0).

12. See next page.

13. It is important to name parent functions in order to classify a wide range of functions according to similar behaviour and characteristics.

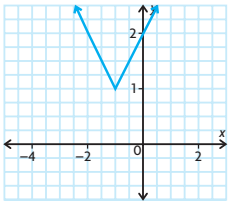
14.



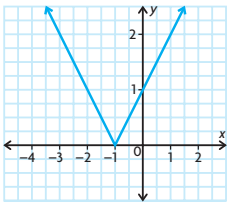
D = $\{x \in \mathbf{R}\}$, R = $\{f(x) \in \mathbf{R}\}$; interval of increase = $(-\infty, \infty)$, no interval of decrease, no discontinuities, x- and y-intercept at (0, 0), odd, $x \to \infty$, $y \to \infty$, and $x \to -\infty$, $y \to -\infty$. It is very similar to $f(x) = x$. It does not, however, have a constant slope.

15. No, cos x is a horizontal translation of sin x.

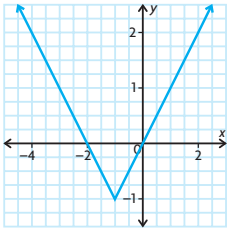
16. The graph can have 0, 1, or 2 zeros.
0 zeros:



1 zero:



2 zeros:

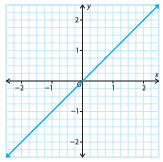
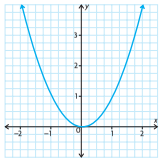
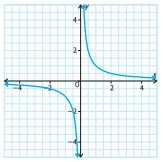
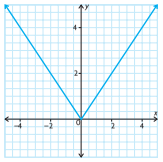
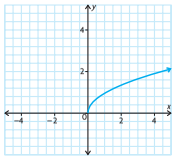
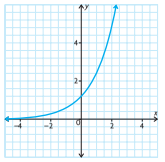
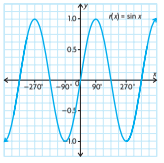


Mid-Chapter Review, p. 28

1. **a)** function; D = {0, 3, 15, 27}, R = {2, 3, 4}
b) function; D = $\{x \in \mathbf{R}\}$, R = $\{y \in \mathbf{R}\}$
c) not a function;
D = $\{x \in \mathbf{R} \mid -5 \le x \le 5\}$,
R = $\{y \in \mathbf{R} \mid -5 \le y \le 5\}$
d) not a function; D = {1, 2, 10}, R = {−1, 3, 6, 7}

2. **a)** Yes. Every element in the domain gets sent to exactly one element in the range.
b) D = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
c) R = {10, 20, 25, 30, 35, 40, 45, 50}

12.

Parent Function	$f(x) = x$	$g(x) = x^2$	$h(x) = \frac{1}{x}$	$k(x) = \lvert x \rvert$	$m(x) = \sqrt{x}$	$p(x) = 2^x$	$r(x) = \sin x$
Sketch							
Domain	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \neq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R} \mid x \geq 0\}$	$\{x \in \mathbf{R}\}$	$\{x \in \mathbf{R}\}$
Range	$\{f(x) \in \mathbf{R}\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \neq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$	$\{f(x) \in \mathbf{R} \mid f(x) > 0\}$	$\{f(x) \in \mathbf{R} \mid -1 \leq f(x) \leq 1\}$
Intervals of Increase	$(-\infty, \infty)$	$(0, \infty)$	None	$(0, \infty)$	$(0, \infty)$	$(-\infty, \infty)$	$[90(4k + 1), 90(4k + 3)]$ $K \in \mathbf{Z}$
Intervals of Decrease	None	$(-\infty, 0)$	$(-\infty, 0)$ $(0, \infty)$	$(-\infty, 0)$	None	None	$[90(4k + 3), 90(4k + 1)]$ $K \in \mathbf{Z}$
Location of Discontinuities and Asymptotes	None	None	$y = 0$ $x = 0$	None	None	$y = 0$	None
Zeros	(0, 0)	(0, 0)	None	(0, 0)	(0, 0)	None	$180k$ $K \in \mathbf{Z}$
y-Intercepts	(0, 0)	(0, 0)	None	(0, 0)	(0, 0)	(0, 1)	(0, 0)
Symmetry	Odd	Even	Odd	Even	Neither	Neither	Odd
End Behaviours	$x \to \infty, y \to \infty$ $x \to -\infty, y \to -\infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to 0$ $x \to -\infty, y \to 0$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to \infty$	$x \to \infty, y \to \infty$	$x \to \infty, y \to \infty$ $x \to -\infty, y \to 0$	Oscillating

3. **a)** D $= \{x \in \mathbf{R}\}$, R $= \{f(x) \in \mathbf{R}\}$; function

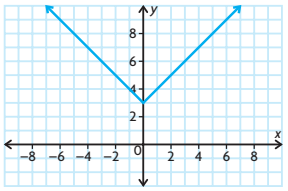
b) D $= \{x \in \mathbf{R} \mid -3 \leq x \leq 3\}$, R $= \{y \in \mathbf{R} \mid -3 \leq y \leq 3\}$; not a function

c) D $= \{x \in \mathbf{R} \mid x \leq 5\}$, R $= \{y \in \mathbf{R} \mid y \geq 0\}$; function

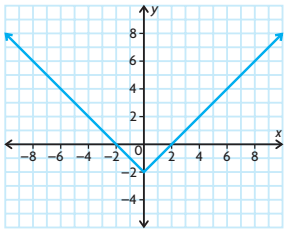
d) D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} \mid y \geq -2\}$; function

4. $-|3|, |0|, |-3|, |-4|, |5|$

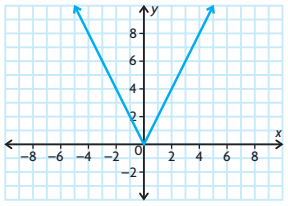
5. **a)**



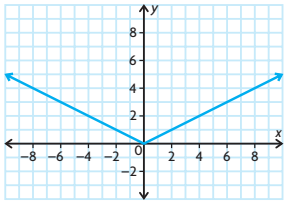
b)



c)



d)



6. **a)** $f(x) = 2^x$

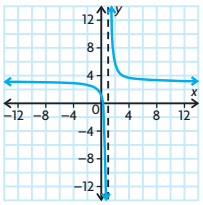
b) $f(x) = \dfrac{1}{x}$

c) $f(x) = \sqrt{x}$

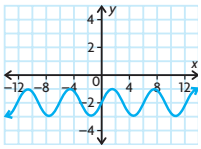
7. **a)** even **c)** neither odd nor even

b) even **d)** neither odd nor even

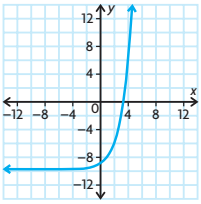
8. **a)** This is $f(x) = \frac{1}{x}$ translated right 1 and up 3; discontinuous



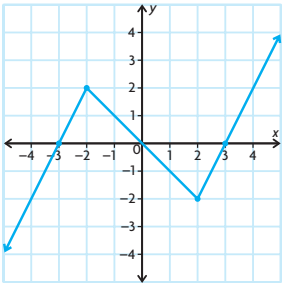
b) This if $f(x) = \sin x$ translated down 2; continuous



c) This is $f(x) = 2^x$ translated down 10; continuous



9.



Lesson 1.4, pp. 35–37

1. **a)** translation 1 unit down

b) horizontal compression by a factor of $\frac{1}{2}$, translation 1 unit right

c) reflection over the x-axis, translation 2 units up, translation 3 units right

d) reflection over the x-axis, vertical stretch by a factor of 2, horizontal compression by a factor of $\frac{1}{4}$

e) reflection over the x-axis, translation 3 units down, reflection over the y-axis, translation 2 units left

f) vertical compression by a factor of $\frac{1}{2}$, translation 6 units up, horizontal stretch by a factor of 4, translation 5 units right

2. **a)** $a = -1, k = \dfrac{1}{2}, d = 0, c = 3$

b) $a = 3, k = \dfrac{1}{2}, d = 0, c = -2$

3. $(2, 3), (1, 3), (1, 6), (1, -6), (-4, -6), (-4, -10)$

4. **a)** $(2, 6), (4, 14), (-2, 10), (-4, 12)$

b) $(5, 3), (7, 7), (1, 5), (-1, 6)$

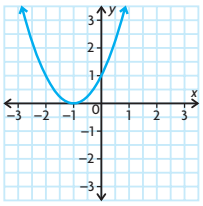
c) $(2, 5), (4, 9), (-2, 7), (-4, 8)$

d) $(1, 0), (3, 4), (-3, 2), (-5, 3)$

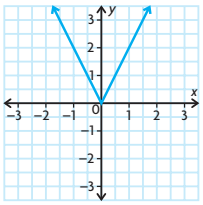
e) $(2, 5), (4, 6), (-2, 3), (-4, 7)$

f) $(1, 2), (2, 6), (-1, 4), (-2, 5)$

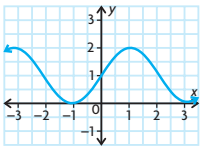
5. **a)** $f(x) = x^2$, translated left 1



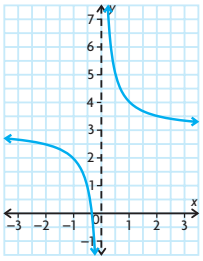
b) $f(x) = |x|$, vertical stretch by 2



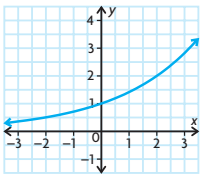
c) $f(x) = \sin x$, horizontal compression of $\frac{1}{3}$, translation up 1



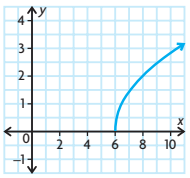
d) $f(x) = \frac{1}{x}$, translation up 3



e) $f(x) = 2^x$, horizontal stretch by 2

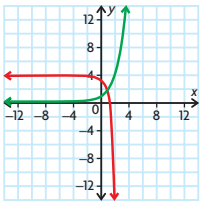


f) $f(x) = \sqrt{x}$, horizontal compression by $\frac{1}{2}$, translation right 6



6. **a)** $D = \{x \in \mathbf{R}\}$,
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$
b) $D = \{x \in \mathbf{R}\}$,
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$
c) $D = \{x \in \mathbf{R}\}$,
$R = \{f(x) \in \mathbf{R} \mid 0 \leq f(x) \leq 2\}$
d) $D = \{x \in \mathbf{R} \mid x \neq 0\}$,
$R = \{f(x) \in \mathbf{R} \mid f(x) \neq 3\}$
e) $D = \{x \in \mathbf{R}\}$,
$R = \{f(x) \in \mathbf{R} \mid f(x) > 0\}$
f) $D = \{x \in \mathbf{R} \mid x \geq 6\}$,
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$

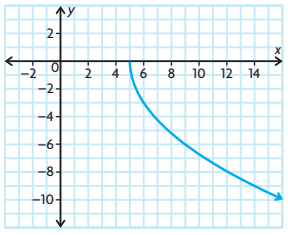
7. **a)**



b) The domain remains unchanged at $D = \{x \in \mathbf{R}\}$. The range must now be less than 4:
$R = \{f(x) \in \mathbf{R} \mid f(x) < 4\}$. It changes from increasing on $(-\infty, \infty)$ to decreasing on $(-\infty, \infty)$. The end behaviour becomes as $x \to -\infty$, $y \to 4$, and as $x \to \infty$, $y \to -\infty$.

c) $g(x) = -2(2^{3(x-1)} + 4)$

8. $y = -3\sqrt{x - 5}$

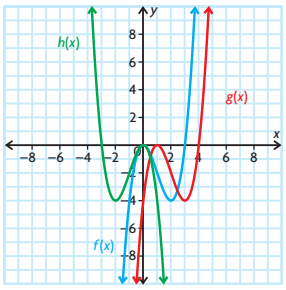


9. **a)** $(3, 24)$ **d)** $(-0.75, -8)$
b) $(-0.5, 4)$ **e)** $(-1, -8)$
c) $(-1, 9)$ **f)** $(-1, 7)$

10. **a)** $D = \{x \in \mathbf{R} \mid x \geq 2\}$,
$R = \{g(x) \in \mathbf{R} \mid g(x) \geq 0\}$
b) $D = \{x \in \mathbf{R} \mid x \geq 1\}$,
$R = \{h(x) \in \mathbf{R} \mid h(x) \geq 4\}$
c) $D = \{x \in \mathbf{R} \mid x \leq 0\}$,
$R = \{k(x) \in \mathbf{R} \mid k(x) \geq 1\}$
d) $D = \{x \in \mathbf{R} \mid x \geq 5\}$,
$R = \{j(x) \in \mathbf{R} \mid j(x) \geq -3\}$

11. $y = 5(x^2 - 3)$ is the same as $y = 5x^2 - 15$, not $y = 5x^2 - 3$.

12.



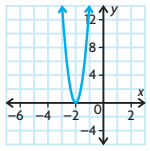
13. **a)** a vertical stretch by a factor of 4
b) a horizontal compression by a factor of $\frac{1}{2}$
c) $(2x)^2 = 2^2x^2 = 4x^2$

14. Answers may vary. For example:

horizontal stretch or compression, based on value of k
↓
vertical stretch or compression, based on value of a
↓
reflection in x-axis if $a < 0$; reflection in y-axis if $k < 0$
↓
horizontal translation based on value of d
↓
vertical translation based on value of c

15. $(4, 5)$

16. **a)** horizontal compression by a factor of $\frac{1}{3}$, translation 2 units to the left
b) because they are equivalent expressions: $3(x + 2) = 3x + 6$
c)



Lesson 1.5, pp. 43–45

1. **a)** $(5, 2)$ **c)** $(-8, 4)$ **e)** $(0, -3)$
b) $(-6, -5)$ **d)** $(2, 1)$ **f)** $(7, 0)$

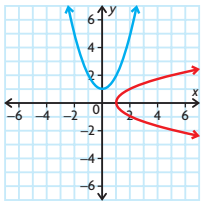
2. **a)** $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$
b) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y \geq 2\}$
c) $D = \{x \in \mathbf{R} \mid x < 2\}$,
$R = \{y \in \mathbf{R} \mid y \geq -5\}$
d) $D = \{x \in \mathbf{R} \mid -5 < x < 10\}$,
$R = \{y \in \mathbf{R} \mid y < -2\}$

3. A and D match; B and F match; C and E match

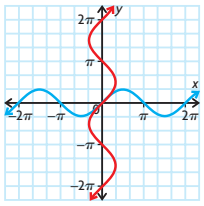
4. **a)** $(4, 129)$
b) $(129, 4)$
c) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$
d) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$
e) Yes; it passes the vertical line test.

5. **a)** $(4, 248)$
b) $(248, 4)$
c) $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R} \mid y \geq -8\}$
d) $D = \{x \in \mathbf{R} \mid x \geq -8\}$ $\mathbf{R} = \{y \in \mathbf{R}\}$
e) No; $(248, 4)$ and $(248, -4)$ are both on the inverse relation.

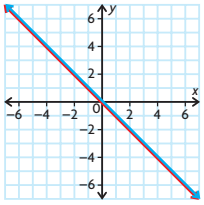
6. **a)** Not a function



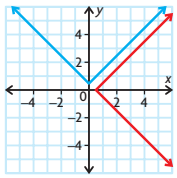
b) Not a function



c) Function



d) Not a function



7. **a)** $C = \frac{5}{9}(F - 32)$; this allows you to convert from Fahrenheit to Celsius.
b) 20 °C = 68 °F

8. **a)** $r = \sqrt{\frac{A}{\pi}}$; this can be used to determine the radius of a circle when its area is known.
b) $A = 25\pi$ cm^2, $r = 5$ cm

9. $k = 2$

10. **a)** 13 **c)** 2 **e)** 1
b) 25 **d)** -2 **f)** $\frac{1}{2}$

11. No; several students could have the same grade point average.

12. **a)** $f^{-1}(x) = \frac{1}{3}(x - 4)$

b) $h^{-1}(x) = -x$

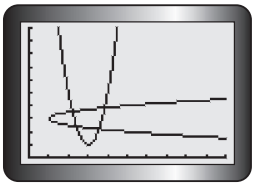
c) $g^{-1}(x) = \sqrt[3]{x + 1}$

d) $m^{-1}(x) = -\frac{x}{2} - 5$

13. **a)** $x = 4(y - 3)^2 + 1$

b) $y = \pm\sqrt{\frac{x - 1}{4}} + 3$

c)



d) (2.20, 3.55), (2.40, 2.40), (3.55, 2.20), (3.84, 3.84)

e) $x \geq 3$ because a negative square root is undefined.

f) $g(2) = 5$, but $g^{-1}(5) = 2$ or 4; the inverse is not a function if this is the domain of g.

14. For $y = -\sqrt{x + 2}$, $D = \{x \in \mathbf{R} \mid x \geq -2\}$ and $R = \{y \in \mathbf{R} \mid y \leq 0\}$. For $y = x^2 - 2$, $D = \{x \in \mathbf{R}\}$ and $R = \{y \in \mathbf{R} \mid y \geq -2\}$. The student would be correct if the domain of $y = x^2 - 2$ is restricted to $D = \{x \in \mathbf{R} \mid x \leq 0\}$.

15. Yes; the inverse of $y = \sqrt{x + 2}$ is $y = x^2 - 2$ so long as the domain of this second function is restricted to $D = \{x \in \mathbf{R} \mid x \geq 0\}$.

16. John is correct.

Algebraic: $y = \frac{x^3}{4} + 2$; $y - 2 = \frac{x^3}{4}$; $4(y - 2) = x^3$; $x = \sqrt[3]{4(y - 2)}$.

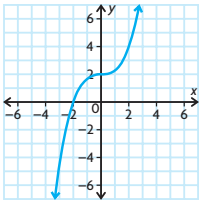
Numeric: Let $x = 4$.

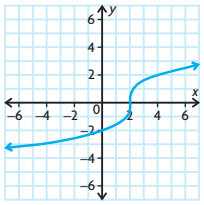
$y = \frac{4^3}{4} + 2 = \frac{64}{4} + 2 = 16 + 2 = 18$;

$x = \sqrt[3]{4(y - 2)} = \sqrt[3]{4(18 - 2)}$

$= \sqrt[3]{4(16)} = \sqrt[3]{64} = 4$.

Graphical:





The graphs are reflections over the line $y = x$.

17. $f(x) = k - x$ works for all $k \in \mathbf{R}$.

$y = k - x$

Switch variables and solve for y: $x = k - y$

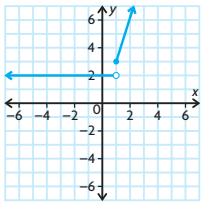
$y = k - x$

So the function is its own inverse.

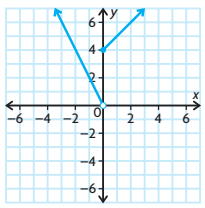
18. If a horizontal line hits the function in two locations, that means there are two points with equal y-values and different x-values. When the function is reflected over the line $y = x$ to find the inverse relation, those two points become points with equal x-values and different y-values, thus violating the definition of a function.

Lesson 1.6, pp. 51–53

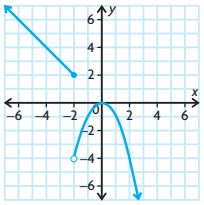
1. **a)**



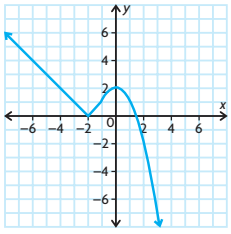
b)



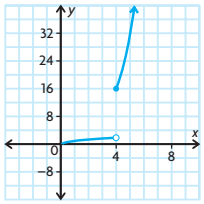
c)



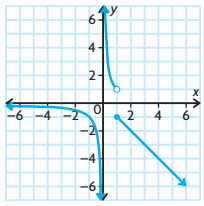
d)



e)



f)



2. **a)** Discontinuous at $x = 1$

b) Discontinuous at $x = 0$

c) Discontinuous at $x = -2$

d) Continuous

e) Discontinuous at $x = 4$

f) Discontinuous at $x = 1$ and $x = 0$

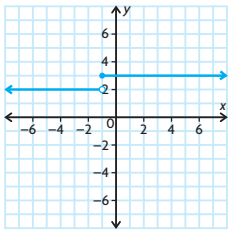
3. **a)** $f(x) = \begin{cases} x^2 - 2, \text{ if } x \leq 1 \\ x + 1, \text{ if } x > 1 \end{cases}$

b) $f(x) = \begin{cases} |x|, \text{ if } x < 1 \\ \sqrt{x}, \text{ if } x \geq 1 \end{cases}$

4. **a)** $D = \{x \in \mathbf{R}\}$; the function is discontinuous at $x = 1$.

b) $D = \{x \in \mathbf{R}\}$; the function is continuous.

5. **a)**

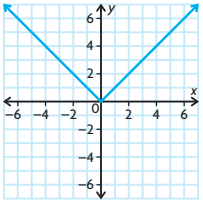


The function is discontinuous at $x = -1$.

$D = \{x \in \mathbf{R}\}$

$R = \{2, 3\}$

b)

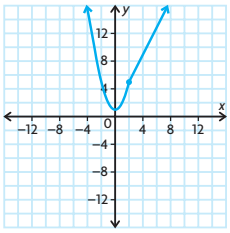


The function is continuous.

$D = \{x \in \mathbf{R}\}$

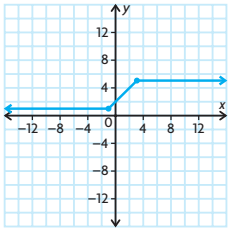
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 0\}$

c)



The function is continuous.
$D = \{x \in \mathbf{R}\}$
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 1\}$

d)

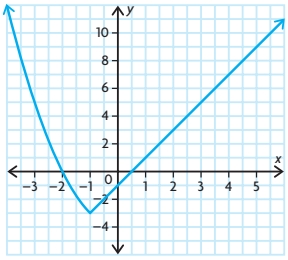


The function is continuous.
$D = \{x \in \mathbf{R}\}$
$R = \{f(x) \in \mathbf{R} \mid 1 \leq f(x) \leq 5\}$

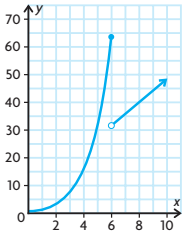
6. $f(x) = \begin{cases} 15, \text{ if } 0 \leq x \leq 500 \\ 15 + 0.02x, \text{ if } x \geq 500 \end{cases}$

7. $f(x) = \begin{cases} 0.35x, \text{ if } 0 \leq x \leq 100\,000 \\ 0.45x - 10\,000, \text{ if } 100\,000 < x \leq 500\,000 \\ 0.55x - 60\,000, \text{ if } x > 500\,000 \end{cases}$

8. $k = 4$

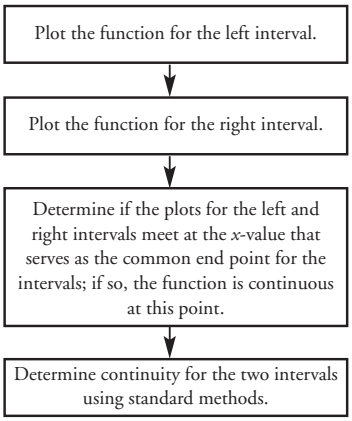


9. a)

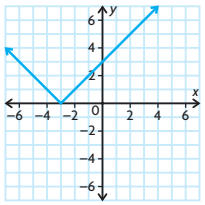


b) The function is discontinuous at $x = 6$.
c) 32 fish
d) $4x + 8 = 64$; $4x = 56$; $x = 14$
e) Answers may vary. For example, three possible events are environmental changes, introduction of a new predator, and increased fishing.

10. Answers may vary. For example:

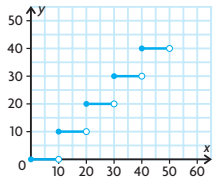


11. $f(x) = |x + 3| = \begin{cases} x + 3, \text{ if } x \geq -3 \\ -x - 3, \text{ if } x < -3 \end{cases}$



12. discontinuous at $p = 0$ and $p = 15$; continuous at $0 < p < 15$ and $p > 15$

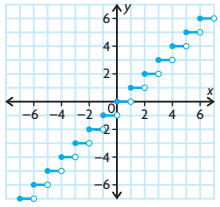
13. $f(x) = \begin{cases} 0, \text{ if } 0 \leq x < 10 \\ 10, \text{ if } 10 \leq x < 20 \\ 20, \text{ if } 20 \leq x < 30 \\ 30, \text{ if } 30 \leq x < 40 \\ 40, \text{ if } 40 \leq x < 50 \end{cases}$



It is often referred to as a step function because the graph looks like steps.

14. To make the first two pieces continuous, $5(-1) = -1 + k$, so $k = -4$. But if $k = -4$, the graph is discontinuous at $x = 3$.

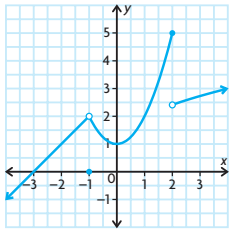
15.



16. Answers may vary. For example:

a) $f(x) = \begin{cases} x + 3, \text{ if } x < -1 \\ x^2 + 1, \text{ if } -1 \leq x \leq 2 \\ \sqrt{x} + 1, \text{ if } x > 2 \end{cases}$

b)



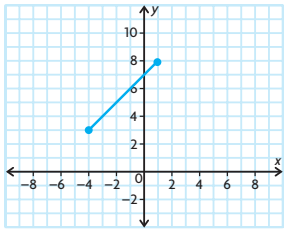
c) The function is not continuous. The last two pieces do not have the same value for $x = 2$.

d) $f(x) = \begin{cases} x + 3, \text{ if } x < -1 \\ x^2 + 1, \text{ if } -1 \leq x \leq 1 \\ \sqrt{x} + 1, \text{ if } x > 1 \end{cases}$

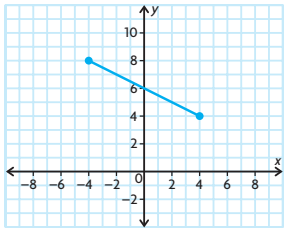
Lesson 1.7, pp. 56–57

1. a) $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
b) $\{(-4, 2), (-2, 3), (1, 1), (4, 2)\}$
c) $\{(-4, -2), (-2, -3), (1, -1), (4, -2)\}$
d) $\{(-4, 8), (-2, 4), (1, 6), (4, 24)\}$

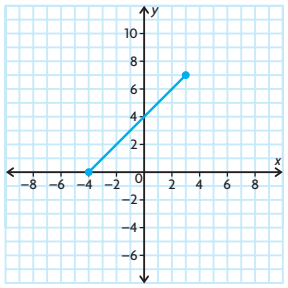
2. a)



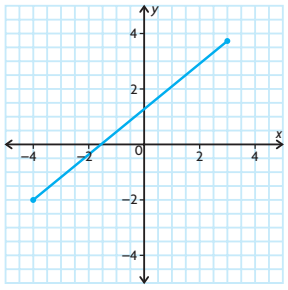
b)



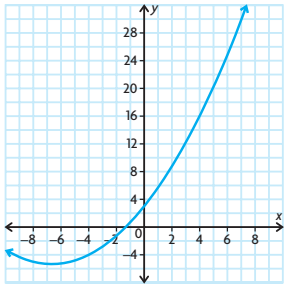
3. a)



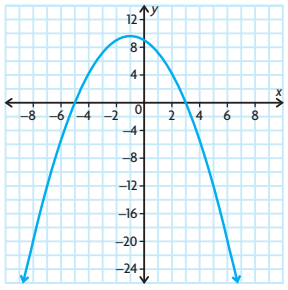
b)



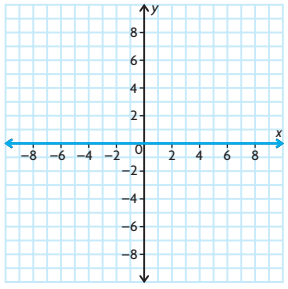
4. a)



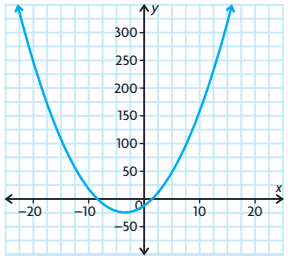
b)



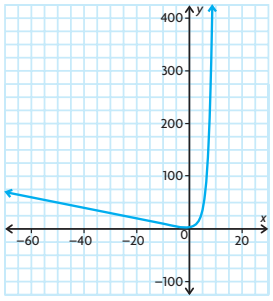
5. a) $y = 0$



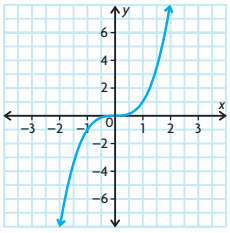
b) $y = x^2 + 7x - 12$



c) $y = |x| + 2^x$

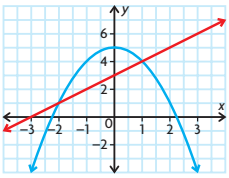


d) $y = x^3$



6. a)–b) Answers may vary. For example, properties of the original graphs such as intercepts and sign at various values of the independent variable figure prominently in the shape of the new function.

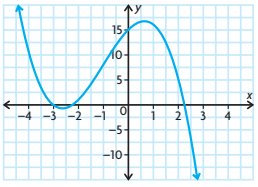
7. a)



b)

x	$f(x)$	$g(x)$	$h(x) = f(x) \times g(x)$
-3	0	-4	0
-2	1	1	1
-1	2	4	8
0	3	5	15
1	4	4	16
2	5	1	5
3	6	-4	-24

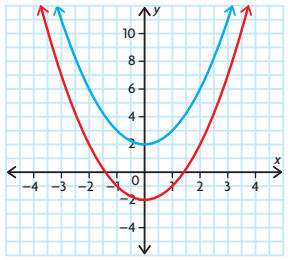
c)



d) $h(x) = (x + 3)(-x^2 + 5)$
$= -x^3 - 3x^2 + 5x + 15$; degree is 3

e) $D = \{x \in \mathbf{R}\}$; this is the same as the domain of both f and g.

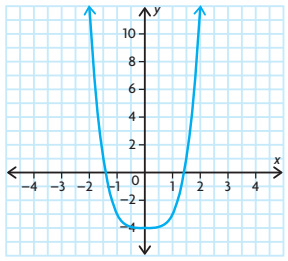
8. a)



b)

x	$f(x)$	$g(x)$	$h(x) = f(x) \times g(x)$
-3	11	7	77
-2	6	2	12
-1	3	-1	-3
0	2	-2	-4
1	3	-1	-3
2	6	2	12
3	11	7	77

c)



d) $h(x) = (x^2 + 2)(x^2 - 2) = x^4 - 4$; degree is 4

e) $D = \{x \in \mathbf{R}\}$

Chapter Review, pp. 60–61

1. a) function; $D = \{x \in \mathbf{R}\}$; $R = \{y \in \mathbf{R}\}$

b) function; $D = \{x \in \mathbf{R}\}$;
$R = \{y \in \mathbf{R} \mid y \leq 3\}$

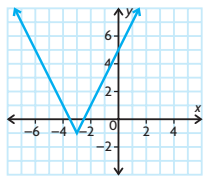
c) not a function;
$D = \{x \in \mathbf{R} \mid -1 \leq x \leq 1\}$;
$R = \{y \in \mathbf{R}\}$

d) function; $D = \{x \in \mathbf{R} \mid x > 0\}$;
$R = \{y \in \mathbf{R}\}$

2. a) $C(t) = 30 + 0.02t$

b) $D = \{t \in \mathbf{R} \mid t \geq 0\}$,
$R = \{C(t) \in \mathbf{R} \mid C(t) \geq 30\}$

3. $D = \{x \in \mathbf{R}\}$,
$R = \{f(x) \in \mathbf{R} \mid f(x) \geq 1\}$



4. $|x| < 2$

5. **a)** Both functions have a domain of all real numbers, but the ranges differ.

b) Both functions are odd but have different domains.

c) Both functions have the same domain and range, but x^2 is smooth and $|x|$ has a sharp corner at (0, 0).

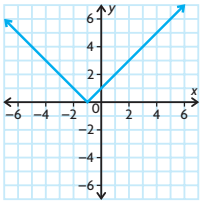
d) Both functions are increasing on the entire real line, but 2^x has a horizontal asymptote while x does not.

6. **a)** Increasing on $(-\infty, \infty)$; odd; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R}\}$

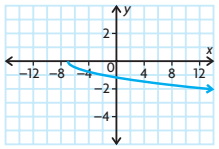
b) Decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$; even; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} \mid f(x) \geq 2\}$

c) Increasing on $(-\infty, \infty)$; neither even nor odd; $D = \{x \in \mathbf{R}\}$; $R = \{f(x) \in \mathbf{R} \mid f(x) > -1\}$

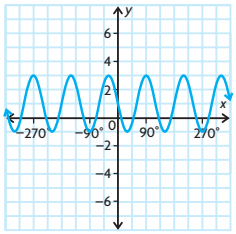
7. **a)** Parent: $y = |x|$; translated left 1



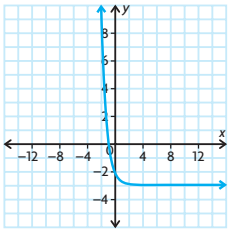
b) Parent: $y = \sqrt{x}$; compressed vertically by a factor of 0.25, reflected across the x-axis, compressed horizontally by a factor of $\frac{1}{3}$, and translated left 7



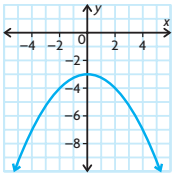
c) Parent: $y = \sin x$; reflected across the x-axis, expanded vertically by a factor of 2, compressed horizontally by a factor of $\frac{1}{3}$, translated up by 1



d) Parent: $y = 2^x$; reflected across the y-axis, compressed horizontally by a factor of $\frac{1}{2}$, and translated down by 3



8. $y = -\left(\frac{1}{2}x\right)^2 - 3$



9. **a)** $(-2, 1)$

b) $(-10, -6)$

c) $(4, 3)$

d) $\left(\frac{17}{5}, 0.3\right)$

e) $(-1, 0)$

f) $(9, -1)$

10. **a)** $(2, 1)$

b) $(-9, -1)$

c) $(7, 0)$

d) $(7, 5)$

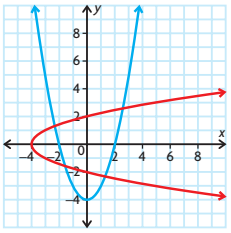
e) $(-3, 0)$

f) $(10, 1)$

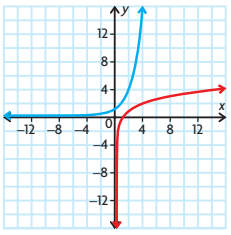
11. **a)** $D = \{x \in \mathbf{R} \mid -2 < x < 2\}$, $R = \{y \in \mathbf{R}\}$

b) $D = \{x \in \mathbf{R} \mid x < 12\}$, $R = \{y \in \mathbf{R} \mid y \geq 7\}$

12. **a)** The inverse relation is not a function.



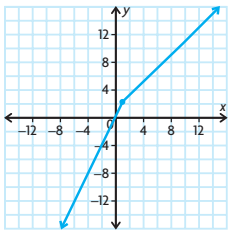
b) The inverse relation is a function.



13. **a)** $f^{-1}(x) = \frac{x - 1}{2}$

b) $g^{-1}(x) = \sqrt[3]{x}$

14.



The function is continuous; $D = \{x \in \mathbf{R}\}$, $R = \{y \in \mathbf{R}\}$

15. $f(x) = \begin{cases} 3x - 1, \text{ if } x \leq 2 \\ -x, \text{ if } x > 2 \end{cases}$;

the function is discontinuous at $x = 2$.

16. In order for $f(x)$ to be continuous at $x = 1$, the two pieces must have the same value when $x = 1$.

When $x = 1$, $x^2 + 1 = 2$ and $3x = 3$.

The two pieces are not equal when $x = 1$, so the function is not continuous at $x = 1$.

17. **a)** $f(x) = \begin{cases} 30, \text{ if } x \leq 200 \\ 24 + 0.03, \text{ if } x > 200 \end{cases}$

b) \$34.50

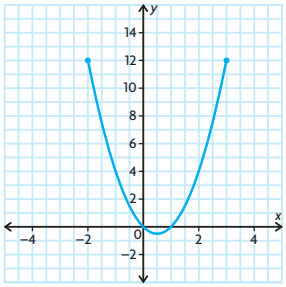
c) \$30

18. **a)** $\{(1, 7), (4, 15)\}$

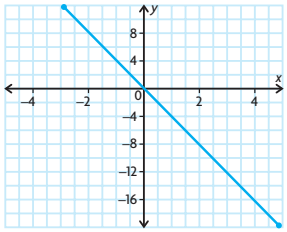
b) $\{(1, -1), (4, -1)\}$

c) $\{(1, 12), (4, 56)\}$

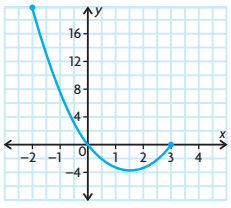
19. **a)**



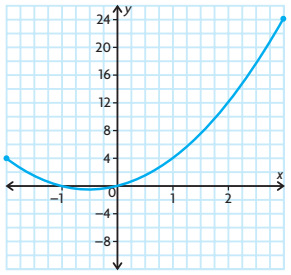
b)



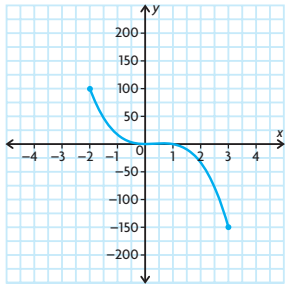
c)



d)



e)

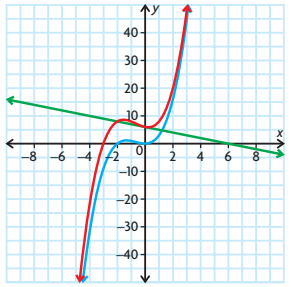


20. **a)** D
b) C
c) A
d) B

21. **a)**

x	-3	-2	-1	0	1	2
$f(x)$	-9	0	1	0	3	16
$g(x)$	9	8	7	6	5	4
$(f+g)(x)$	0	8	8	6	8	20

b)–c)



d) $x^3 + 2x^2 - x + 6$

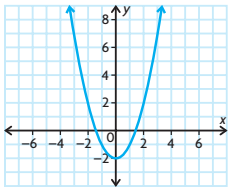
e) Answers may vary. For example, (0, 0) belongs to f, (0, 6) belongs to g and (0, 6) belongs to $f + g$. Also, (1, 3) belongs to f, (1, 5) belongs to g and (1, 8) belongs to $f + g$.

Chapter Self-Test, p. 62

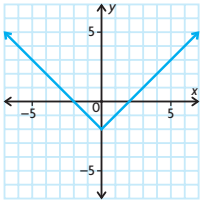
1. **a)** Yes. It passes the vertical line test.
b) D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R} | y \geq 0\}$

2. **a)** $f(x) = x^2$ or $f(x) = |x|$

b)



or

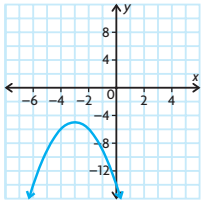


c) The graph was translated 2 units down.

3. $f(-x) = |3(-x)| + (-x)^2$
$= |3x| + x^2 = f(x)$

4. 2^x has a horizontal asymptote while x^2 does not. The range of 2^x is $\{y \in \mathbf{R} \mid y > 0\}$ while the range of x^2 is $\{y \in \mathbf{R} \mid y \geq 0\}$. 2^x is increasing on the whole real line and x^2 has an interval of decrease and an interval of increase.

5. reflection over the x-axis, translation down 5 units, translation left 3 units



6. horizontal stretch by a factor of 2, translation 1 unit up; $f(x) = \text{if } |\frac{1}{2}x| + 1$

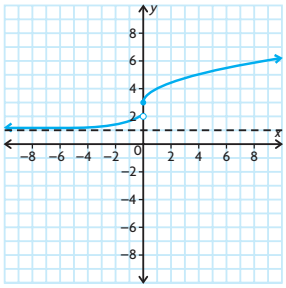
7. **a)** $(-4, 17)$
b) (5, 3)

8. $f^{-1}(x) = -\frac{x}{2} - 1$

9. **a)** \$9000

b) $f(x) = \begin{cases} 0.05, \text{ if } x \leq 50\,000 \\ 0.12x - 6000, \text{ if } x > 50\,000 \end{cases}$

10. **a)**



b) $f(x)$ is discontinuous at $x = 0$ because the two pieces do not have the same value when $x = 0$. When $x = 0$, $2^x + 1 = 2$ and $\sqrt{x} + 3 = 3$.

c) Intervals of increase: $(-\infty, 0)$, $(0, \infty)$; no intervals of decrease

d) D $= \{x \in \mathbf{R}\}$,
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Chapter 2

Getting Started, p. 66

1. **a)** $\frac{4}{3}$ **b)** $-\frac{6}{7}$

2. **a)** Each successive first difference is 2 times the previous first difference. The function is exponential.
b) The second differences are all 6. The function is quadratic.

3. **a)** $-\frac{3}{2}, 2$ **b)** 0 **c)** 45°, 225° **d)** $-270°, -90°$

4. **a)** vertical compression by a factor of $\frac{1}{2}$
b) vertical stretch by a factor of 2, horizontal translation 4 units to the right
c) vertical stretch by a factor of 3, reflection across x-axis, vertical translation 7 units up
d) vertical stretch by a factor of 5, horizontal translation 3 units to the right, vertical translation 2 units down,

5. **a)** $A = 1000(1.08)^t$
b) \$1259.71
c) No, since the interest is compounded each year, each year you earn more interest than the previous year.

6. **a)** 15 m; 1 m
b) 24 s
c) 15 m

7.

Linear relations
constant; same as slope of line; positive for lines that slope up from left to right; negative for lines that slope down from left to right; 0 for horizontal lines.

Rates of Change

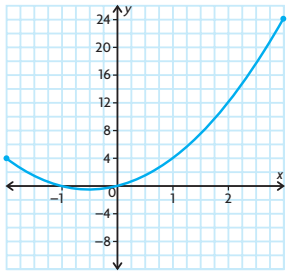
Nonlinear relations
variable; can be positive, negative, or 0 for different parts of the same relation

Lesson 2.1, pp. 76–78

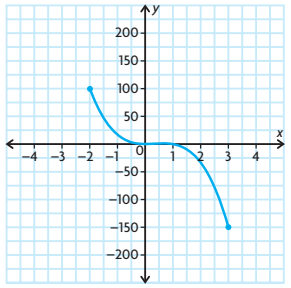
1. **a)** 19 **b)** 15 **c)** 13 **d)** 12 **e)** 11.4 **f)** 11.04

2. **a)** **i)** 15 m/s **ii)** -5 m/s

d)



e)



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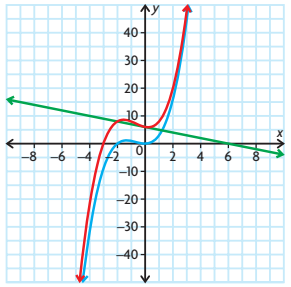

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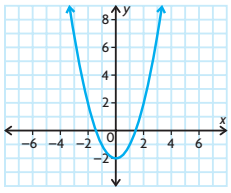

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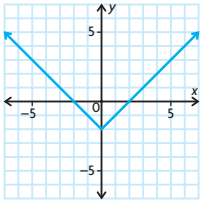

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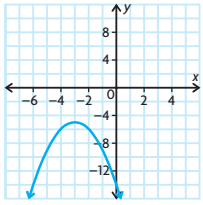

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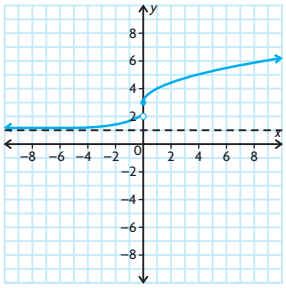

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Chapter 2

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constant; same as slope of line; positive for lines that slope up from left to right; negative for lines that slope down from left to right; 0 for horizontal lines.

Rates of Change

Nonlinear relations
variable; can be positive, negative, or 0 for different parts of the same relation

Lesson 2.1, pp. 76–78

1. **a)** 19 **b)** 15 **c)** 13 **d)** 12 **e)** 11.4 **f)** 11.04

2. **a)** **i)** 15 m/s **ii)** -5 m/s

b) During the first interval, the height is increasing at 15 m/s; during the second interval, the height is decreasing at 5 m/s.

3. $f(x)$ is always increasing at a constant rate. $g(x)$ is decreasing on $(-\infty, 0)$ and increasing on $(0, \infty)$, so the rate of change is not constant.

4. **a)** 352, 138, 286, 28, 60, −34 people/h
b) the rate of growth of the crowd at the rally
c) A positive rate of growth indicates that people were arriving at the rally. A negative rate of growth indicates that people were leaving the rally.

5. **a)** 203, 193, 165, 178.5, 218.5, 146 km/day
b) No. Some days the distance travelled was greater than others.

6. 4; 4; the average rate of change is always 4 because the function is linear, with a slope of 4.

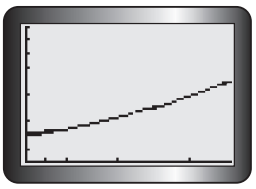
7. The rate of change is 0 for 0 to 250 min. After 250 min, the rate of change is \$0.10/min.

8. **a)** **i)** 750 people/year
ii) 3000 people/year
iii) 12 000 people/year
iv) 5250 people/year
b) No; the rate of growth increases as the time increases.
c) You must assume that the growth continues to follow this pattern, and that the population will be 5 120 000 people in 2050.

9. −2 m/s

10. **a)** **i)** \$2.60/sweatshirt
ii) \$2.00/sweatshirt
iii) \$1.40/sweatshirt
iv) \$0.80/sweatshirt
b) The rate of change is still positive, but it is decreasing. This means that the profit is still increasing, but at a decreasing rate.
c) No; after 6000 sweatshirts are sold, the rate of change becomes negative. This means that the profit begins to decrease after 6000 sweatshirts are sold.

11. **a)**



b) The rate of change will be greater farther in the future. The graph is getting steeper as the values of t increase.

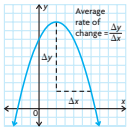
c) **i)** 1500 people/year
ii) 1700 people/year
iii) 2000 people/year
iv) 2500 people/year
d) The prediction was correct.

12. Answers may vary. For example:
a) Someone might calculate the average increase in the price of gasoline over time. One might also calculate the average decrease in the price of computers over time.
b) An average rate of change might be useful for predicting the behaviour of a relationship in the future.
c) An average rate of change is calculated by dividing the change in the dependent variable by the corresponding change in the dependent variable.

13. −7.8%

14. Answers may vary. For example:

AVERAGE RATE OF CHANGE

Definition in your own words	Personal example	Visual representation
the change in one quantity divided by the change in a related quantity	I record the number of miles I run each week versus the week number. Then, I can calculate the average rate of change in the distance I run over the course of weeks.	

15. 80 km/h

Lesson 2.2, pp. 85–88

1. **a)**

Preceding Interval	$\Delta f(x)$	Δx	Average Rate of Change, $\frac{\Delta f(x)}{\Delta x}$
$1 \le x \le 2$	$13 - (-2) = 15$	$2 - 1 = 1$	15
$1.5 \le x \le 2$	8.75	0.5	17.5
$1.9 \le x \le 2$	1.95	0.1	19.5
$1.99 \le x \le 2$	0.1995	0.01	19.95

Following Interval	$\Delta f(x)$	Δx	Average Rate of Change, $\frac{\Delta f(x)}{\Delta x}$
$2 \le x \le 3$	$38 - 13 = 25$	$3 - 2 = 1$	25
$2 \le x \le 2.5$	11.25	0.5	22.5
$2 \le x \le 2.1$	2.05	0.1	20.5
$2 \le x \le 2.01$	0.2005	0.01	20.05

b) 20

2. **a)** 5.4 m/s **b)** 5.4 m/s
c) Answers may vary. For example: I prefer the centred interval method. Fewer calculations are required, and it takes into account points on each side of the given point in each calculation.

3. **a)** 200
b) 40 raccoons/month
c) 50 raccoons/month
d) The three answers represent different things: the population at a particular time, the average rate of change prior to that time, and the instantaneous rate of change at that time.

4. **a)** −24 **b)** 0 **c)** 48 **d)** 96

5. −27 m/s

6. \$11 610 per year

7. **a)** 0 people/year
b) Answers may vary. For example: Yes, it makes sense. It means that the populations in 2000 and 2024 are the same, so their average rate of change is 0.
c) The average rate of change from 2000 to 2012 is 18 000 people/year; the average rate of change from 2012 to 2024 is −18 000 people/year.
d) $t = 12$

8. About −\$960 per year; when the car turns five, it loses \$960 of its value.

9. **a)** 1.65 s **b)** about 14 m/s

10. 100π cm^3/cm

11. If David knows how far he has travelled and how long he has been driving, he can calculate his average speed from the beginning of the trip by dividing the distance travelled by the time he has been driving.

12. **a)** −22.5 °F/min
b) Answers may vary. For example: −25.5 °F/min
c) Answers may vary. For example, the first rate is using a larger interval to estimate the instantaneous rate.
d) Answers may vary. For example, the second estimate is better, as it uses a much smaller interval to estimate the instantaneous rate.

13. Answers may vary. For example:

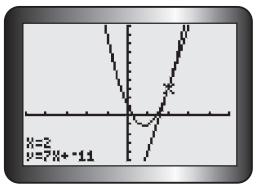
Method of Estimating Instantaneous Rate of Change	Advantage	Disadvantage
series of preceding intervals and following intervals	accounts for differences in the way that change occurs on either side of the given point	must do two sets of calculations
series of centred intervals	accounts for points on either side of the given interval in same calculation	to get a precise answer, numbers involved will need to have several decimal places
difference quotient	more precise	calculations can be tedious or messy

14. **a)** 100π cm^2/cm
b) 240π cm^2/cm
15. 36 cm^2/cm
16. 160π cm^2/cm

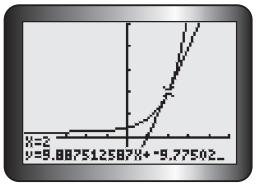
Lesson 2.3, pp. 91–92

1. **a)** about 7 **c)** about 0.25
b) about 10 **d)** 2

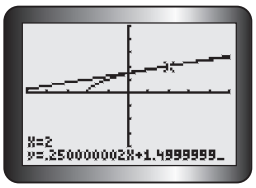
2. **a)**



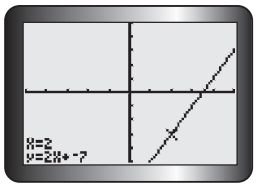
b)



c)

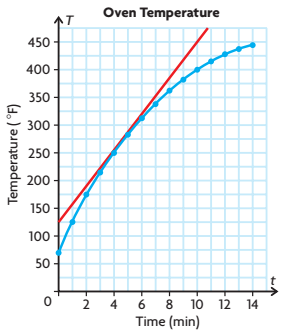


d)



3. **a)** Set A: 0, 0, 0, 0
Set B: 14, 1.4, 5, 0.009
Set C: -4, -0.69, -3, -0.009
b) Set A: All slopes are zero.
Set B: All slopes are positive.
Set C: All slopes are negative.

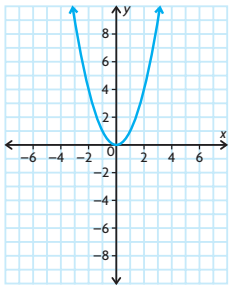
4. **a)** and **b)**



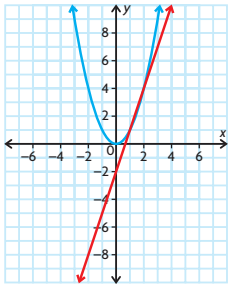
c) 31
d) Rate of change is about 30 °F/min at $x = 5$.
e) Answers may vary. For example: The two answers are about the same. The slope of the tangent line at the point is the same as the instantaneous rate of change at the point.

5. Answers may vary. For example: Similarity: the calculation; difference: average rate of change is over an interval; instantaneous rate of change is at a point.

6. **a)**



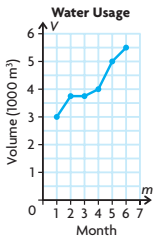
b)



c) (1.5, 2.25)

Mid-Chapter Review, p. 95

1. **a)**



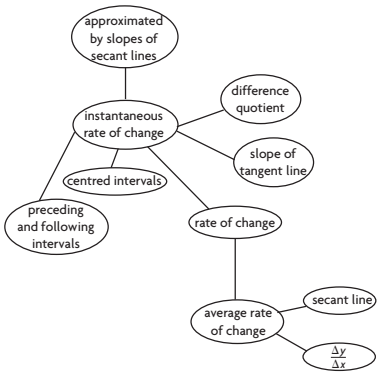
b) 750; 250; 1100; 400 m^3/month
c) April and May
d) 580 m^3/month

2. **a)** The equation models exponential growth. This means that the average rate of change between consecutive years will always increase.
b) The instantaneous rate of change in population in 2010 is about 950 people per year.

3. **a)** 10 m/s; -10 m/s
b) $t = 2$; Answers may vary. For example: The graph has a vertex at (2, 21). It appears that a tangent line at this point would be horizontal.
$\frac{(f(2.01) - f(1.99))}{0.02}$

4. 0.9 m/day

5. Answers may vary. For example:



6. Answers may vary. For example:

Points	Slope of Secant
(2, 9) and (1, 2)	7
(2, 9) and (1.5, 4.375)	9.25
(2, 9) and (1.9, 7.859)	11.41
(2, 9) and (2.1, 10.261)	12.61
(2, 9) and (2.5, 16.625)	15.25
(2, 9) and (3, 28)	19

The slope of the tangent line at (2, 9) is about 12.

7. 4

8. The instantaneous rate of change of the function whose graph is shown is 4 at $x = 2$.

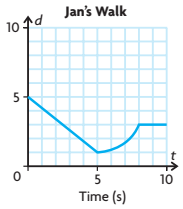
9. Answers may vary. For example:
a) 0 **b)** 4 **c)** 5 **d)** 8

Lesson 2.4, pp. 103–106

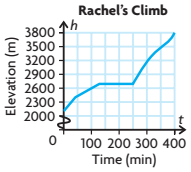
1. **a)** C **b)** A **c)** B

2. All of the graphs show that the speed is constant. In a), the speed is positive and constant. In b), the speed is negative and constant. In c), the speed is 0, which is constant.

3.

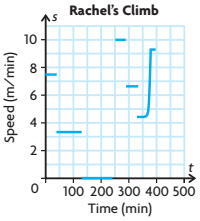


4. **a)** Answers may vary. For example:

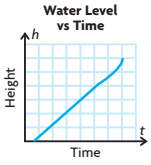


b) Average speed over first 40 min is 7.5 m/min, average speed over next 90 min is 3.3 m/min, average speed over next 120 min is 0 m/min, average speed over next 40 min is 10 m/min, average speed over next 45 min is 6.7 m/min, and average speed over last 60 min is 5.7 m/min.

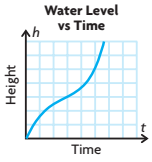
c) Answers may vary. For example:



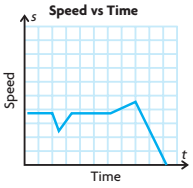
5. **a)** Answers may vary. For example:



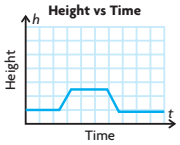
b) Answers may vary. For example:



6. **a)** Answers may vary. For example:



b) Answers may vary. For example:

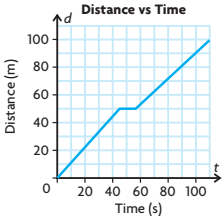


7. **a)** 1.11 m/s

b) 0.91 m/s

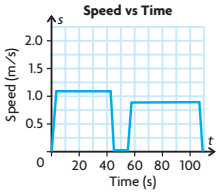
c) The graph of the first length would be steeper, indicating a quicker speed. The graph of the second length would be less steep, indicating a slower speed.

d) Answers may vary. For example:



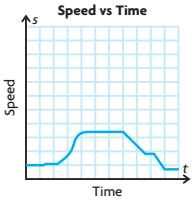
e) 0 m/s

f) Answers may vary. For example:



8. **a)** A **b)** C **c)** D **d)** B

9. Answers may vary. For example:

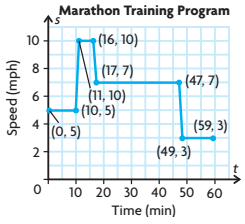


10. **a)** and **b)**

i) Start 5 m from sensor. Walk toward sensor at a constant rate of 1 m/s for 3 s. Walk away from sensor at a constant rate of 1 m/s for 3 s.

ii) Start 6 m from sensor. Walk toward sensor at a constant rate of 1 m/s for 2 s. Stand still for 1 s. Walk toward sensor at a constant rate of 1 m/s for 2 s. Walk away from sensor at a constant rate of 1.5 m/s.

11. **a)** Answers may vary. For example:



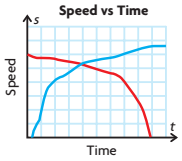
b) 5 mph/min

c) −0.1842 mph/min

d) The answer to part c) is an average rate of change over a long period, but the runner does not slow down at a constant rate during this period.

12. Answers may vary. For example: Walk from (0, 0) to (5, 5) and stop for 5 s. Then run to (15, 30). Continue walking to (20, 5) and end at (25, 0). What is the maximum speed and minimum speed on an interval? Create the speed versus time graph from these data.

13. Answers may vary. For example:



14. If the original graph showed an increase in rate, it would mean that the distance travelled during each successive unit of time would be greater—meaning a graph that curves upward. If the original graph showed a straight, horizontal line, then it would mean that the distance travelled during each successive unit of time would be greater—meaning a steady increasing straight line on the second graph. If the original graph showed a decrease in rate, it would mean that the distance travelled during each successive unit of time would be less—meaning a line that curves down.

Lesson 2.5, pp. 111–113

1. Answers may vary. For example, I used the difference quotient when $a = 1.5$ and $h = 0.001$ and got an estimate for the instantaneous rate of change in cost that was close to 0.

2. 0

3. **a)** The slopes of the tangent lines are positive, but close to 0.

b) The slopes of the tangent lines are negative, but close to 0.

4. **a)** The slopes of the tangent lines are negative, but close to 0.

b) The slopes of the tangent lines are positive, but close to 0.

5. **a)** The slope is 0.

b) The slope is 0.

c) The slope is 0.

d) The slope is 0.

6. **a)** minimum

b) maximum

c) minimum

d) maximum

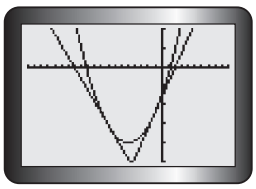
e) maximum

f) maximum

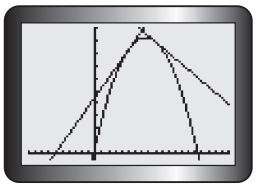
7. $t = 2.75$; Answers may vary. For example: The slopes of tangents for values of t less than about 2.75 would be positive, while slopes of tangents for values of t greater than about 2.75 would be negative.

8. **a)** $x = -5$; minimum
$x = 7.5$; maximum
$x = 3.25$; minimum
$x = 6$; maximum

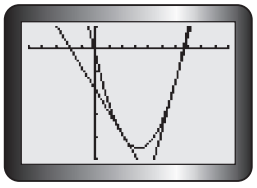
b) i)



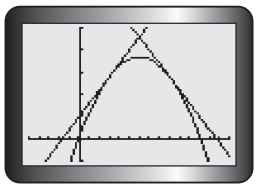
ii)



iii)



iv)



c) Answers may vary. For example, if the sign of the slope of the tangent changed from positive to negative, there was a maximum. If the sign of the slope of the tangent changed from negative to positive, there was a minimum.

9. **a)** **i)** maximum $= (0, 100)$;
minimum $= (5, 44.4)$
ii) maximum $= (10, 141.6)$;
minimum $= (0, 35)$

b) For an equation that represents exponential growth (where $r > 0$), the minimum value will always be at point a and the maximum value will always be at point b, because y will always increase as x increases. For an equation that represents exponential decay (where $r < 0$), the minimum value will always be at point b and the maximum value will always be at point a, because y will always decrease as x increases.

10. Answers may vary. For example, the slope of the tangent at 0.5 s is 0. The slope of the tangent at 0 s is 5, and the slope of the tangent at 1 s is -5. So, the diver reaches her maximum height at 0.5 s.

11. Answers may vary. For example, yes, this observation is correct. The slope of the tangent at 1.5 s is 0. The slopes of the tangents between 1 s and 1.5 s are negative, and the slopes of the tangent lines between 1.5 s and 2 s are positive. So, the minimum of the function occurs at 1.5 s.

12. Answers may vary. For example, estimate the slope of the tangent line to the curve when $x = 5$ by writing an equation for the slope of any secant line on the graph of $R(x)$. If the slope of the tangent is 0, this will confirm there may be a maximum at $x = 5$. If the slopes of tangent lines to the left are positive and the slopes of tangent lines to the right are negative, this will confirm that a maximum occurs at $x = 5$.

13. Answers may vary. For example, because $\sin 90°$ gives a maximum value of 1, I know that a maximum occurs when $(k(x - d)) = 90°$. Solving this equation for x will tell me what types of x-values will give a maximum. For example, when $k = 2$ and $d = 3$,
$(2(x - 3°)) = 90°$
$(x - 3°) = 45°$
$x = 48°$

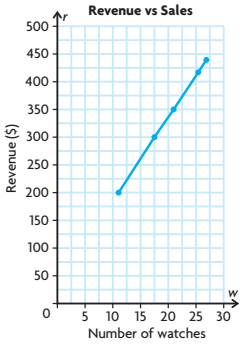
14. Myra is plotting (instantaneous) velocity versus time. The rates of change Myra calculates represent acceleration. When Myra's graph is increasing, the car is accelerating. When Myra's graph is decreasing, the car is decelerating. When Myra's graph is constant, the velocity of the car is constant; the car is neither accelerating nor decelerating.

15. $-4, -2, 4, 6$; The rule appears to be "multiply the x-coordinate by 2." 12, 3, 12, 27; The rule for $f(x) = x^3$ seems to be "square the x-coordinate and multiply by 3."

Chapter Review, pp. 116–117

1. **a)** Yes. Divide revenue by number of watches, and the slope is 17.5.

b) Answers may vary. For example:



The data represent a linear relationship.

c) \$17.50 per watch

d) \$17.50; this is the slope of the line on the graph.

2. **a)** 1.5 m/s

b) -1.5 m/s

c) The time intervals have the same length. The amount of change is the same, but with opposite signs for the two intervals. So, the rates of change are the same for the two intervals, but with opposite signs.

3. **a)** $E = 2500m + 10\,000$

b) \$2500 per month

c) No; the equation that represents this situation is linear, and the rate of change over time for a linear equation is constant.

4. **a)** Answers may vary. For example, because the unit of the equation is years, you would not choose $3 \le t \le 4.25$ and $4 \le t \le 5$. A better choice would be $3.75 \le t \le 4.0$ and $4.0 \le t \le 4.25$.

b) Answers may vary. For example, find the average of the two interval values: $\frac{(600.56 + 621.91)}{2} = \$611.24$

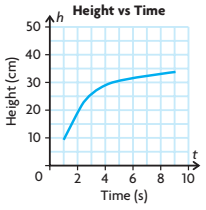
5. **a)** Answers may vary. For example, squeezing the interval.

b) 4.19 cm/s

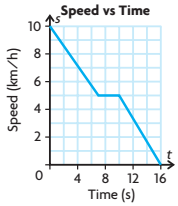
6. **a)** -2 **b)** 0 **c)** 4

7. **a)** -37 **b)** -17 **c)** 0 **d)** 23

8. Answers may vary. For example:



9. **a)** Answers may vary. For example:

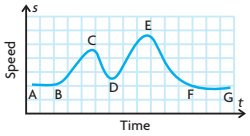


b) $-\frac{5}{7}$ km/h/s

c) From (7, 5) to (12, $\frac{10}{3}$), the rate of change of speed in $-\frac{1}{3}$ km/h/s

d) $-\frac{5}{6}$ km/h/s

10. The roller coaster moves at a slow steady speed between A and B. At B, it begins to accelerate as it moves down to C. Going uphill from C to D it decelerates. At D, it starts to move down and accelerates to E, where the speed starts to decrease until F, where it maintains a slower speed to G, the end of the track.



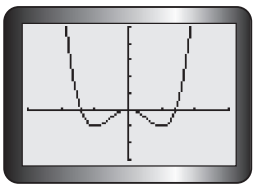
11. **a)** minimum **d)** minimum
b) maximum **e)** minimum
c) maximum **f)** maximum

12. **a)** **i)** $m = h - 26$ **ii)** $m = -4h - 48$
b) **i)** $m = -26$ **ii)** $m = -48$

13. **a)** To the left of a maximum, the instantaneous rates of change are positive. To the right, the instantaneous rates of change are negative.

b) To the left of a minimum, the instantaneous rates of change are negative. To the right, the instantaneous rates of change are positive.

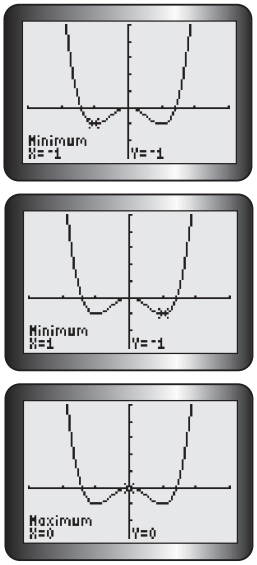
14. **a)**



b) minimum: $x = -1, x = 1$
maximum: $x = 0$

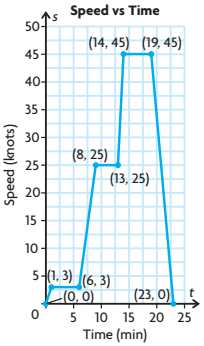
c) The slopes of tangent lines for points to the left of a minimum will be negative, while the slopes of tangent lines for points to the right of a minimum will be positive. The slopes of tangent lines for points to the left of a maximum will be positive, while the slopes of tangent lines for points to the right of a minimum will be negative.

d)



Chapter Self-Test, p. 118

1. **a)**



b) 11 kn/min; 0 kn/min; the two different average rates of change indicate that the boat was increasing its speed from $t = 6$ to $t = 8$ at a rate of 11knots/min and moving at a constant speed from $t = 8$ to $t = 13$.

c) 11 kn/min

2. **a)** -1

b) The hot cocoa is cooling by 1 °C/min on average.

c) -0.75

d) The hot cocoa is cooling by 0.75 °C/min after 30 min.

e) The rate decreases over the interval, until it is nearly 0 and constant.

3. **a)** \$310 per dollar spent

b) $-$\$100 per dollar spent

c) The positive sign for part a) means that the company is increasing its profit when it spends between \$8000 and \$10 000 on advertising. The negative sign means the company's profit is decreasing when it spends \$50 000 on advertising.

4. **a)** -1; 0 (minimum); 7

b) 4.5; -4.5; 0 (maximum)

Chapter 3

Getting Started, p. 122

1. **a)** $6x^3 - 22x^2$

b) $x^2 + 2x - 24$

c) $24x^3 - 44x^2 - 40x$

d) $5x^3 + 31x^2 - 68x + 32$

2. **a)** $(x + 7)(x - 4)$

b) $2(x - 2)(x - 7)$

3. **a)** $x = -6$

b) $x = -3, 4.5$

c) $x = -3, -8$

d) $x = \frac{1}{3}, -4$

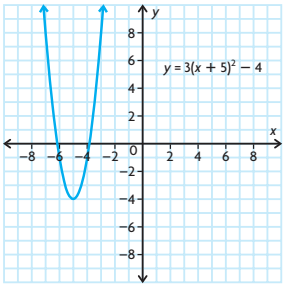
4. **a)** vertical compression by a factor of $\frac{1}{4}$; horizontal translation 3 units to the right; vertical translation 9 units up

b) vertical compression by a factor of $\frac{1}{4}$; vertical translation 7 units down

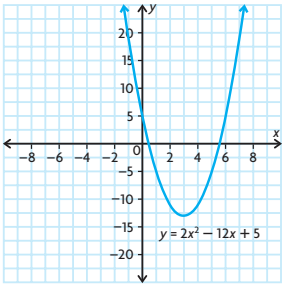
5. **a)** $y = 2(x - 5)^2 - 2$

b) $y = -2x^2 + 3$

6. **a)**



b)



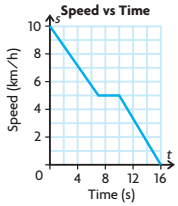
7. **a)** quadratic

b) other

c) other

d) linear

9. **a)** Answers may vary. For example:

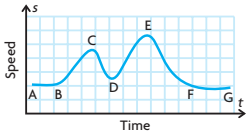


b) $-\frac{5}{7}$ km/h/s

c) From (7, 5) to (12, $\frac{10}{3}$), the rate of change of speed in $-\frac{1}{3}$ km/h/s

d) $-\frac{5}{6}$ km/h/s

10. The roller coaster moves at a slow steady speed between A and B. At B, it begins to accelerate as it moves down to C. Going uphill from C to D it decelerates. At D, it starts to move down and accelerates to E, where the speed starts to decrease until F, where it maintains a slower speed to G, the end of the track.



11. **a)** minimum **d)** minimum
b) maximum **e)** minimum
c) maximum **f)** maximum

12. **a)** **i)** $m = h - 26$ **ii)** $m = -4h - 48$
b) **i)** $m = -26$ **ii)** $m = -48$

13. **a)** To the left of a maximum, the instantaneous rates of change are positive. To the right, the instantaneous rates of change are negative.

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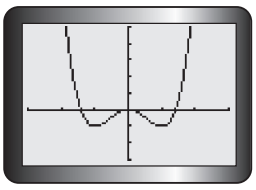
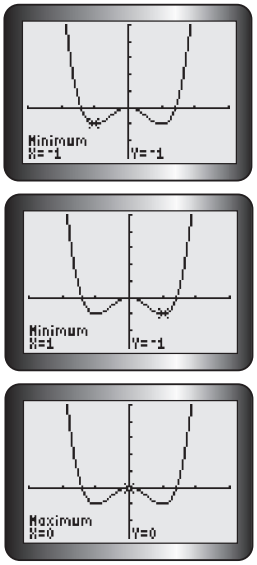
14. **a)**



b) minimum: $x = -1, x = 1$
maximum: $x = 0$

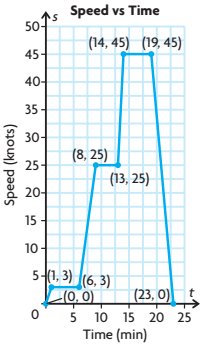
c) The slopes of tangent lines for points to the left of a minimum will be negative, while the slopes of tangent lines for points to the right of a minimum will be positive. The slopes of tangent lines for points to the left of a maximum will be positive, while the slopes of tangent lines for points to the right of a minimum will be negative.

d)



Chapter Self-Test, p. 118

1. **a)**



b) 11 kn/min; 0 kn/min; the two different average rates of change indicate that the boat was increasing its speed from $t = 6$ to $t = 8$ at a rate of 11knots/min and moving at a constant speed from $t = 8$ to $t = 13$.

c) 11 kn/min

2. **a)** -1

b) The hot cocoa is cooling by 1 °C/min on average.

c) -0.75

d) The hot cocoa is cooling by 0.75 °C/min after 30 min.

e) The rate decreases over the interval, until it is nearly 0 and constant.

3. **a)** \$310 per dollar spent

b) $-$\$100 per dollar spent

c) The positive sign for part a) means that the company is increasing its profit when it spends between \$8000 and \$10 000 on advertising. The negative sign means the company's profit is decreasing when it spends \$50 000 on advertising.

4. **a)** -1; 0 (minimum); 7

b) 4.5; -4.5; 0 (maximum)

Chapter 3

Getting Started, p. 122

1. **a)** $6x^3 - 22x^2$

b) $x^2 + 2x - 24$

c) $24x^3 - 44x^2 - 40x$

d) $5x^3 + 31x^2 - 68x + 32$

2. **a)** $(x + 7)(x - 4)$

b) $2(x - 2)(x - 7)$

3. **a)** $x = -6$

b) $x = -3, 4.5$

c) $x = -3, -8$

d) $x = \frac{1}{3}, -4$

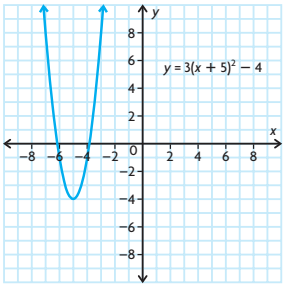
4. **a)** vertical compression by a factor of $\frac{1}{4}$; horizontal translation 3 units to the right; vertical translation 9 units up

b) vertical compression by a factor of $\frac{1}{4}$; vertical translation 7 units down

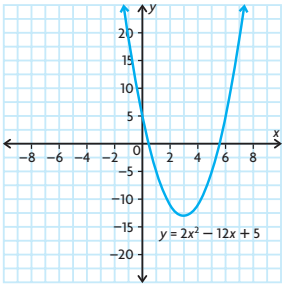
5. **a)** $y = 2(x - 5)^2 - 2$

b) $y = -2x^2 + 3$

6. **a)**



b)



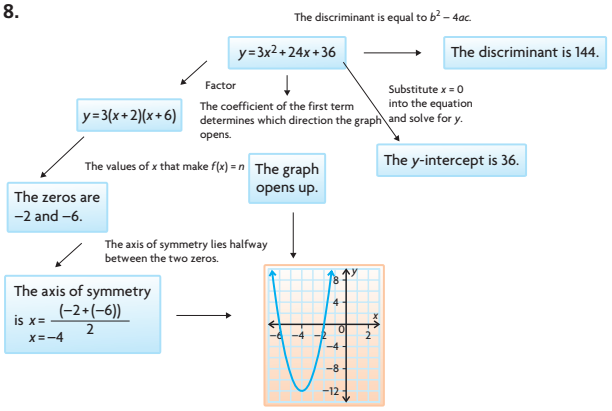
7. **a)** quadratic

b) other

c) other

d) linear

8.



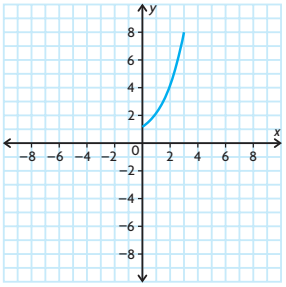
Lesson 3.1, pp. 127–128

1. **a)** This represents a polynomial function because the domain is the set of all real numbers, the range does not have a lower bound, and the graph does not have horizontal or vertical asymptotes.
b) This represents a polynomial function because the domain is the set of all real numbers, the range is the set of all real numbers, and the graph does not have horizontal or vertical asymptotes.
c) This is not a polynomial function because it has a horizontal asymptote.
d) This represents a polynomial function because the domain is the set of all real numbers, the range does not have an upper bound, and the graph does not have horizontal or vertical asymptotes.
e) This is not a polynomial function because its domain is not all real numbers.
f) This is not a polynomial function because it is a periodic function.

2. **a)** polynomial; the exponents of the variables are all natural numbers
b) polynomial; the exponents of the variables are all natural numbers
c) polynomial; the exponents of the variables are all natural numbers
d) other; the variable is under a radical sign
e) other; the function contains another function in the denominator
f) polynomial; the exponents of the variables are all natural numbers

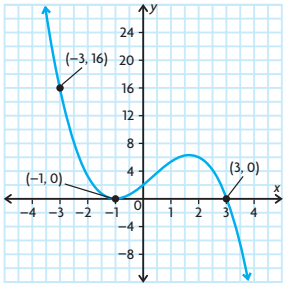
3. **a)** linear **c)** linear
b) quadratic **d)** cubic

4.



a) The graph looks like one half of a parabola, which is the graph of a quadratic equation.
b) There is a variable in the exponent.

5.



6. Answers may vary. For example, any equation of the form $y = a\left(-\frac{4}{3}x^2 + \frac{8}{3}x + 4\right)$ will have the same zeros, but have a different y-intercept and a different value for $f(-3)$. Any equation of the form $y = x\left(-\frac{4}{3}x^2 + \frac{8}{3}x + 4\right)$ would have two of the same zeros, but a different value for $f(-3)$ and different positive/negative intervals.

7. $y = x + 5, y = x^2 + 5, y = x^3 + 5, y = x^4 + 5$

8. Answers may vary. For example:

Definition	**Characteristics**
A polynomial is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number.	The domain of the function is all real numbers, but the range can have restrictions; except for polynomial functions of degree zero (whose graphs are horizontal lines), the graphs of polynomials do not have horizontal or vertical asymptotes. The shape of the graph depends on its degree.
Polynomials	
Examples $x^2 + 4x + 6$	**Non-Examples** $\sqrt{x + 1}$

Lesson 3.2, pp. 136–138

1. **a)** 4; −4; as $x \to +/-\infty, y \to -\infty$
b) 5; 2; as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$
c) 3; −3; as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$
d) 4; 24; as $x \to +/-\infty, y \to \infty$

2. **a)** Turning points
a) minimum 1, maximum 3
b) minimum 0, maximum 4
c) minimum 0, maximum 2
d) minimum 1, maximum 3
b) Zeros
a) minimum 0, maximum 4
b) minimum 1, maximum 5
c) minimum 1, maximum 3
d) minimum 0, maximum 4

3. **i)** **a)** The degree is even.
b) The leading coefficient is negative.
ii) **a)** The degree is even.
b) The leading coefficient is negative.
iii) **a)** The degree is odd.
b) The leading coefficient is negative.
iv) **a)** The degree is even.
b) The leading coefficient is positive.
v) **a)** The degree is odd.
b) The leading coefficient is negative.
vi) **a)** The degree is odd.
b) The leading coefficient is positive.

4. **a)** as $x \to +/-\infty, y \to \infty$
b) as $x \to -\infty, y \to \infty$ and as $x \to \infty, y \to -\infty$
c) as $x \to -\infty, y \to -\infty$ and as $x \to \infty, y \to \infty$
d) as $x \to +/-\infty, y \to -\infty$

e) as $x \to +/-\infty$, $y \to \infty$

f) as $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to -\infty$

5. **a)** D: The graph extends from quadrant III to quadrant I and the y-intercept is 2.

b) A: The graph extends from quadrant III to quadrant IV.

c) E: The graph extends from quadrant II to quadrant I and the y-intercept is -5.

d) C: The graph extends from quadrant II to quadrant I and the y-intercept is 0.

e) F: The graph extends from quadrant II to quadrant IV.

f) B: The graph extends from quadrant III to quadrant I and the y-intercept is 1.

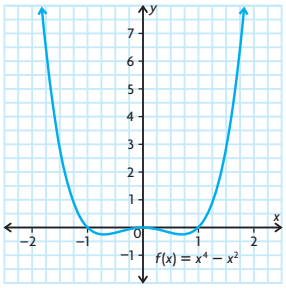
6. **a)** Answers may vary. For example, $f(x) = 2x^3 + 5$.

b) Answers may vary. For example, $f(x) = 6x^2 + x - 4$.

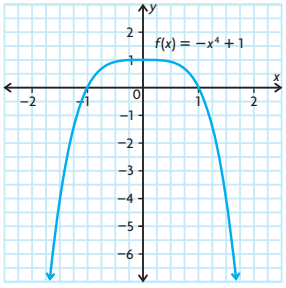
c) Answers may vary. For example, $f(x) = -x^4 - x^3 + 7$.

d) Answers may vary. For example, $f(x) = -9x^5 + x^4 - x^3 - 2$.

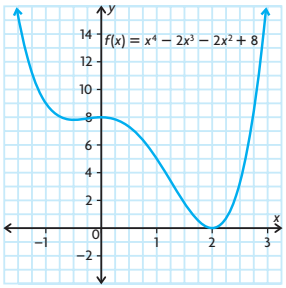
7. **a)** Answers may vary. For example:



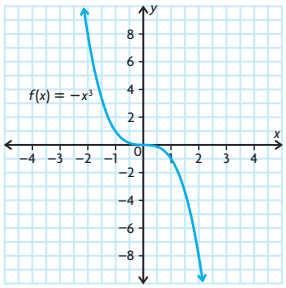
b) Answers may vary. For example:



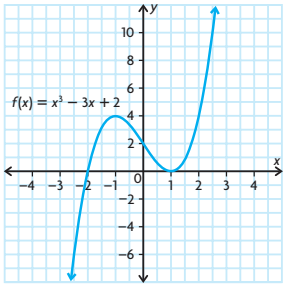
c) Answers may vary. For example:



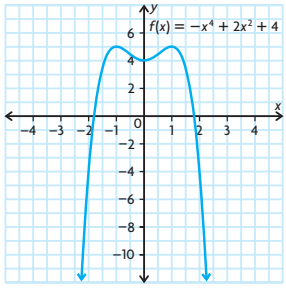
d) Answers may vary. For example:



e) Answers may vary. For example:



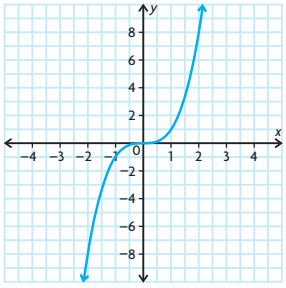
f) Answers may vary. For example:



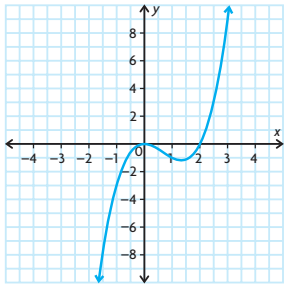
8. An odd-degree polynomial can have only local maximums and minimums because the y-value goes to $-\infty$ and ∞ at each end of the function. An even-degree polynomial can have absolute maximums and minimums because it will go to either $-\infty$ at both ends or ∞ at both ends of the function.

9. even number of turning points

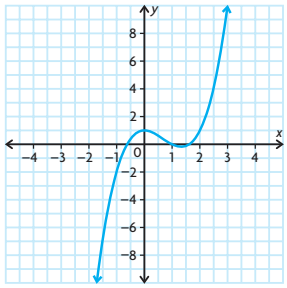
10. **a)** Answers may vary. For example: $f(x) = x^3$



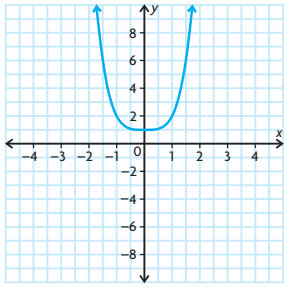
b) Answers may vary. For example: $f(x) = x^3 - 2x^2$



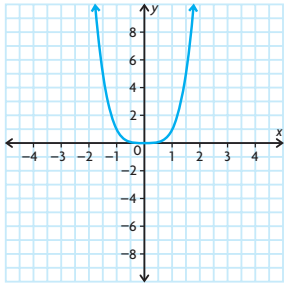
c) Answers may vary. For example: $f(x) = x^3 - 2x^2 + 1$



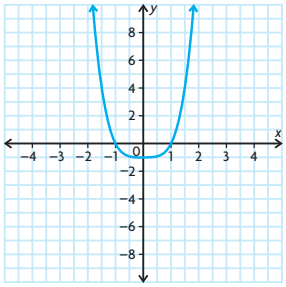
11. **a)** Answers may vary. For example: $f(x) = x^4 + 1$



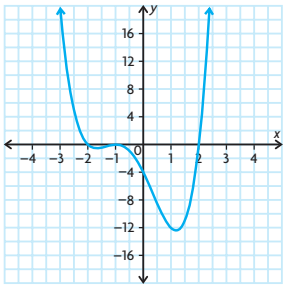
b) Answers may vary. For example: $f(x) = x^4$



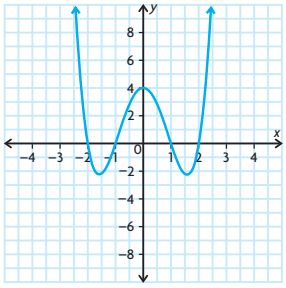
c) Answers may vary. For example:
$f(x) = x^4 - 1$



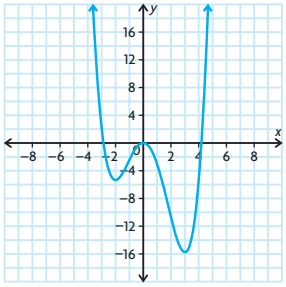
d) Answers may vary. For example:
$f(x) = x^4 + 2x^3 - 3x^2 - 8x - 4$



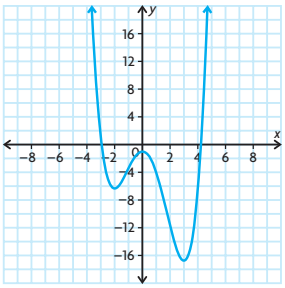
e) Answers may vary. For example:
$f(x) = x^4 - 5x^2 + 4$



12. **a)** Answers may vary. For example:
$f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2$



and $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - 3x^2 - 1$



b) zero and leading coefficient of the function

13. **a)** 700 people

b) The population will decrease because the leading coefficient is negative.

14. **a)** False; Answers may vary. For example, $f(x) = x^2 + x$ is not an even function.

b) True

c) False; Answers may vary. For example, $f(x) = x^2 + 1$ has no zeros.

d) False; Answers may vary. For example, $f(x) = -x^2$ has end behaviour opposite the behaviour stated.

15. Answers may vary. For example, "What are the turning points of the function?", "What is the leading coefficient of the function?", and "What are the zeros of the function?" If the function has 0 turning points or an even number of turning points, then it must extend to the opposite side of the x-axis. If it has an odd number of turning points, it must extend to the same side of the x-axis. If the leading coefficient is known, it can be determined exactly which quadrants the function extends to/from and if the function has been vertically stretched. If the zeros are known, it can be determined if the function has been vertically translated up or down.

16. **a)** $b = 0$

b) $b = 0, d = 0$

Lesson 3.3, pp. 146–148

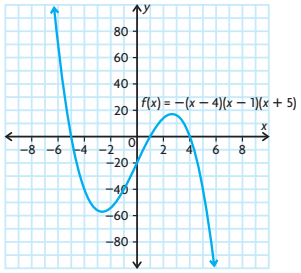
1. **a)** C: The graph has zeros of -1 and 3, and it extends from quadrant III to quadrant I.

b) A: The graph has zeros of -1 and 3, and it extends from quadrant II to quadrant III.

c) B: The graph has zeros of -1 and 3, and it extends from quadrant II to quadrant IV.

d) D: The graph has zeros of -1, 0, 3, and 5, and it extends from quadrant II to quadrant I.

2. **a)**



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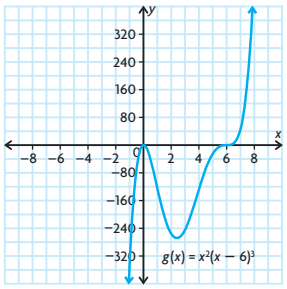
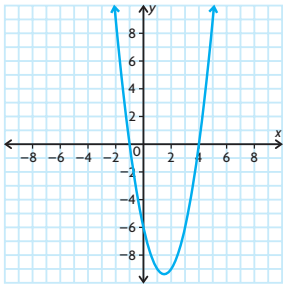


3. **a)** $f(x) = k(x + 1)(x - 4)$;
$f(x) = 4(x + 1)(x - 4)$;
$f(x) = -2(x + 1)(x - 4)$

b) $f(x) = \frac{3}{2}(x + 1)(x - 4)$

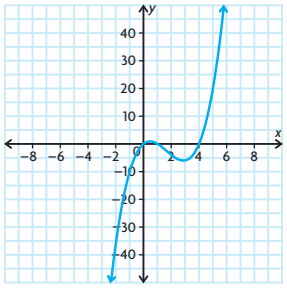


4. **a)** $y = 0.5(x + 3)(x - 2)(x - 5)$

b) $y = -(x + 1)^2(x - 2)(x - 4)$

5. Family 1: A, G, I
Family 2: B, E
Family 3: C, F, H, K
Family 4: D, J, L

6. **a)**



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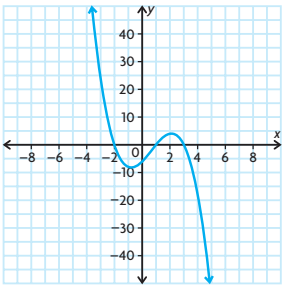
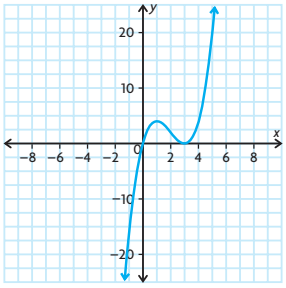
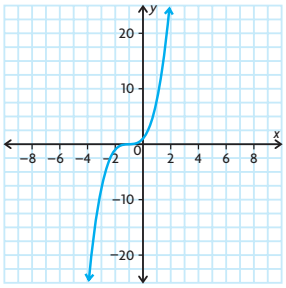


c)



d)



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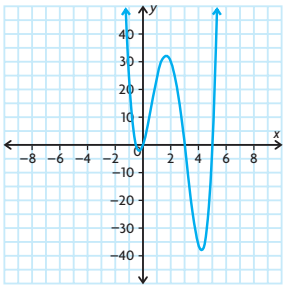

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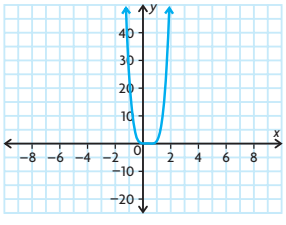
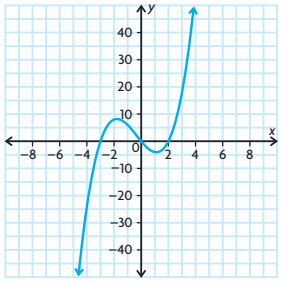
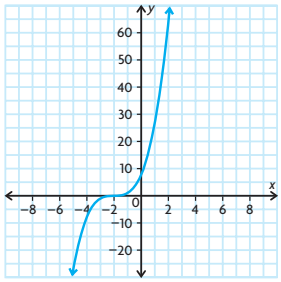


7. a) Answers may vary. For example:

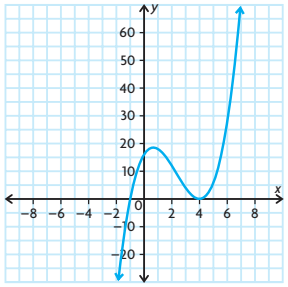
i) $y = x(x + 3)(x - 2)$



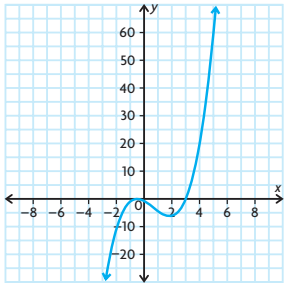
ii) $y = (x + 2)^3$



iii) $y = (x + 1)(x - 4)^2$



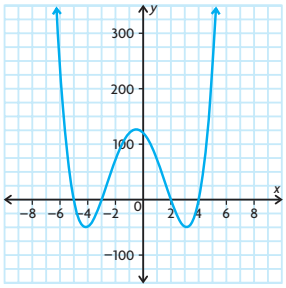
iv) $y = (x - 3)\left(x + \frac{1}{2}\right)^2$



b) No, as all the functions belong to a family of equations.

8. Answers may vary. For example:

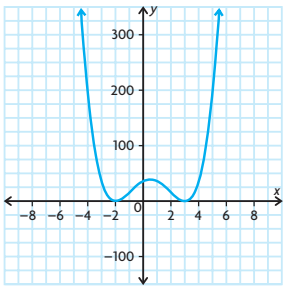
a) $y = (x + 5)(x + 3)(x - 2)(x - 4)$



$y = 2(x + 5)(x + 3)(x - 2)(x - 4)$

$y = -5(x + 5)(x + 3)(x - 2)(x - 4)$

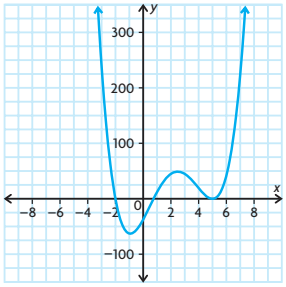
b) $y = (x + 2)^2(x - 3)^2$



$y = 10(x + 2)^2(x - 3)^2$

$y = 7(x + 2)^2(x - 3)^2$

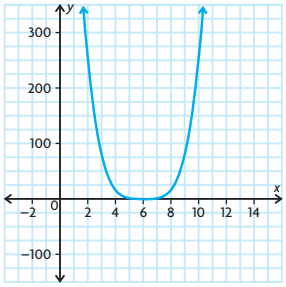
c) $y = (x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$



$y = -(x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$

$y = \frac{2}{5}(x + 2)\left(x - \frac{3}{4}\right)(x - 5)^2$

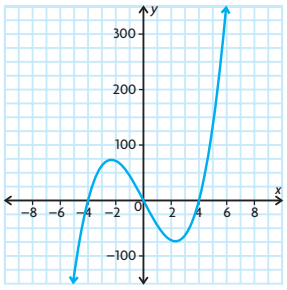
d) $y = (x - 6)^4$



$y = 15(x - 6)^4$

$y = -3(x - 6)^4$

9. **a)**



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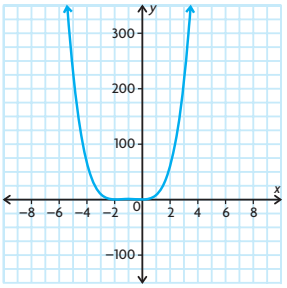

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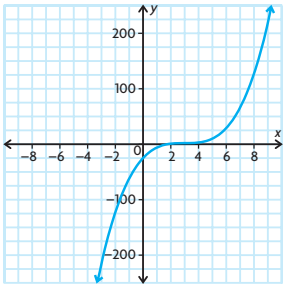
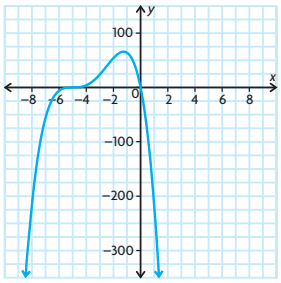
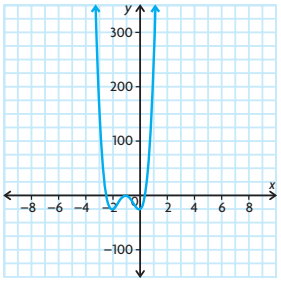


d)

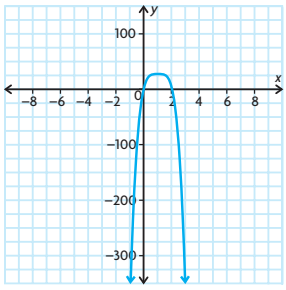


10. Answers may vary. For example:

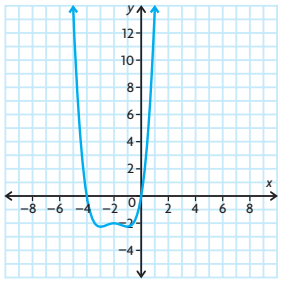
a)



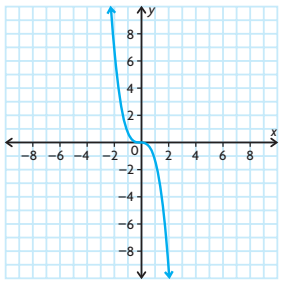
b)



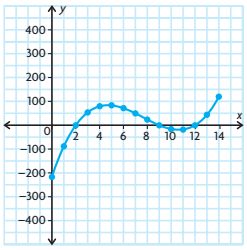
c)



d)



11. **a)**



b) $y = (x - 2)(x - 9)(x - 12)$

c) No; $\{x \in \mathbf{R} | 0 \leq x \leq 14\}$

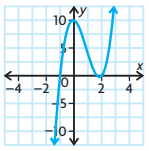
12. **a)** $y = x^3 + 2x^2 - x - 2$

b) $y = -\frac{2}{5}(x - 1)(x + 2)(x + 4)$

13. **a)** $f(x) = -6(x + 3)(x + 5)$

b) $f(x) = 2(x + 2)(x - 3)(x - 4)$

14. $k = 3$



The zeros are $\frac{5}{3}$, -1, and 2.

$f(x) = (3x - 5)(x + 1)(x - 2)$

15. **a)** It has zeros at 2 and 4, and it has turning points at 2, 3, and 4. It extends from quadrant II to quadrant I.

b) It has zeros at -4 and 3, and it has turning points at $-\frac{5}{3}$ and 3. It extends from quadrant III to quadrant I.

16. **a)** 832 cm^3

b) 2.93 cm by 24.14 cm by 14.14 cm or 5 cm by 20 cm by 10 cm

c) $0 < x < 10$; The values of x are the side lengths of squares that can be cut from the sheet of cardboard to produce a box with positive volume. Since the sheet of cardboard is 30 cm by 20 cm, the side lengths of a square cut from each corner have to be less than 10 cm, or an entire edge would be cut away, leaving nothing to fold up.

d) The square that is cut from each corner must be larger than 0 cm by 0 cm but smaller than 10 cm by 10 cm.

Lesson 3.4, pp. 155–158

1. **a)** B: $y = x^3$ has been vertically stretched by a factor of 2, horizontally translated 3 units to the right, and vertically translated 1 unit up.

b) C: $y = x^3$ has been reflected in the x-axis, vertically compressed by a factor of $\frac{1}{3}$, horizontally translated 1 unit to the left, and vertically translated 1 unit down.

c) A: $y = x^4$ has been vertically compressed by a factor of 0.2, horizontally translated 4 units to the right, and vertically translated 3 units down.

d) D: $y = x^4$ has been reflected in the x-axis, vertically stretched by a factor of 1.5, horizontally translated 3 units to the left, and vertically translated 4 units up.

2. **a)** $y = x^4$; vertical stretch by a factor of $\frac{5}{4}$ and vertical translation of 3 units up

b) $y = x$; vertical stretch by a factor of 3 and vertical translation of 4 units down

c) $y = x^3$; horizontal compression by a factor of $\frac{1}{3}$, horizontal translation of $\frac{4}{3}$ units to the left, and vertical translation of 7 units down

d) $y = x^4$; reflection in the x-axis and horizontal translation of 8 units to the left

e) $y = x^2$; reflection in the x-axis, vertical stretch by a factor of 4.8, and horizontal translation 3 units left

f) $y = x^3$; vertical stretch by a factor of 2, horizontal stretch by a factor of 5, horizontal translation of 7 units to the left, and vertical translation of 4 units down

3. a) $y = x^3$ has been translated 3 units to the left and 4 units down.

$y = (x + 3)^3 - 4$

b) $y = x^4$ has been reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 4 units to the left, and vertically translated 5 units up.

$y = -2(x + 4)^4 + 5$

c) $y = x^4$ has been vertically compressed by a factor of $\frac{1}{4}$, horizontally translated 1 unit to the right, and vertically translated 2 units down.

$y = \frac{1}{4}(x - 1)^4 - 2$

d) $y = x^3$ has been reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 3 units to the right, and vertically translated 4 units down.

$y = -2(x - 3)^3 - 4$

4. a) vertically stretched by a factor of 12, horizontally translated 9 units to the right, and vertically translated 7 units down

b) horizontally stretched by a factor of $\frac{8}{7}$, horizontally translated 1 unit to the left, and vertically translated 3 units up

c) vertically stretched by a factor of 2, reflected in the x-axis, horizontally translated 6 units to the right, and vertically translated 8 units down

d) horizontally translated 9 units to the left

e) reflected in the x-axis, vertically stretched by a factor of 2, reflected in the y-axis, horizontally compressed by a factor of $\frac{1}{3}$, horizontally translated 4 units to the right, and vertically translated 5 units down

f) horizontally stretched by a factor of $\frac{4}{3}$ and horizontally translated 10 units to the right

5. a) $y = 8x^2 - 11$

$y = x^2$ was vertically stretched by a factor of 8 and vertically translated 11 units down.

b) $y = -\frac{1}{4}x^2 + 1.25$

$y = x^2$ was reflected in the x-axis, vertically compressed by a factor of $\frac{1}{4}$, and vertically translated 1.25 units up.

6. a) $\left(-6\frac{1}{5}, -\frac{1}{2}\right), (-6, 0), \left(-5\frac{3}{5}, 4\right)$

b) $(2, 2), (0, 3), (-4, 11)$

c) $\left(3, 2\frac{1}{2}\right), \left(4, -\frac{1}{2}\right), \left(6, -24\frac{1}{2}\right)$

d) $\left(-7, -2\frac{1}{10}\right), \left(0, -2\right), \left(14, -1\frac{1}{5}\right)$

e) $\left(1, 1\frac{9}{10}\right), \left(0, \frac{9}{10}\right), \left(-2, -7\frac{1}{10}\right)$

f) $(-11, -8), (-4, -7), (10, 1)$

7. $y = -\frac{1}{4}(x - 1)^4 + 3$

8. $(-2, 8), (0, 0), (2, -8)$

9. a) -2 and -4

b) 4

c) -3 and 1

d) no x-intercepts

e) 6.68 and 9.32

f) -3.86

10. a) 1; $0 = 2(x - 4)^3 + 1$ has only one solution.

b) 0; $0 = 2(x - 4)^4 + 1$ has no solution.

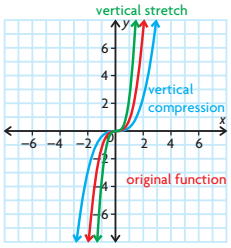
c) 1 when n is odd, since an odd root results in only one value; 0 when n is even, since there is no value for an even root of a negative number.

11. a) The reflection of the function $y = x^n$ in the x-axis will be the same as its reflection in the y-axis for odd values of n.

b) The reflections will be different for even values of n. The reflection in the x-axis will be $y = -x^n$, and the reflection in the y-axis will be $y = (-x)^n$. For odd values of n, $-x^n$ equals $(-x^n)$. For even values of n, $-x^n$ does not equal $(-x^n)$.

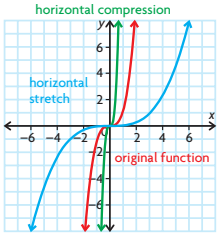
12. a) Vertical stretch and compression:

$y = ax^3$



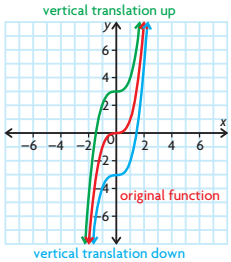
Horizontal stretch and compression:

$y = (kx)^3$



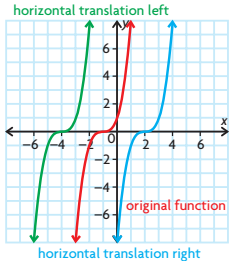
Vertical translation up or down:

$y = x^3 + c$

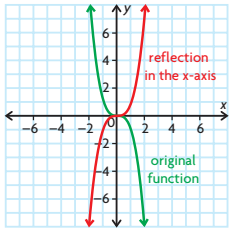


Horizontal translation left or right:

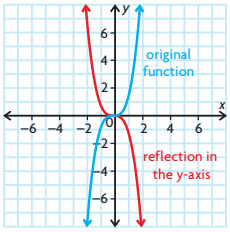
$y = (x - d)^3$



Reflection in the x-axis: $y = -x^3$



Reflection in the y-axis: $y = (-x)^3$



b) When using a table of values to sketch the graph of a function, you may not select a large enough range of values for the domain to produce an accurate representation of the function.

13. Yes, you can. The zeros of the first function have the same spacing between them as the zeros of the second function. Also, the ratio of the distances of the two curves above or below the x-axis at similar distances between the zeros is always the same. Therefore, the two curves have the same general shape, and one can be transformed into the other.

14. $y = (x - 1)^2 (x + 1)^2$ has zeroes at $x = \pm 1$ where the x-axis is tangent to these points. $y = 2(x - 1)^2(x + 1)^2 + 1$ is obtained by vertically stretching the original function by a factor of 2 and vertically translating up 1 unit. This results in a new graph that has no zeroes.

15. $f(x) = 5(2(x + 3))^2 + 1$

Mid-Chapter Review, p. 161

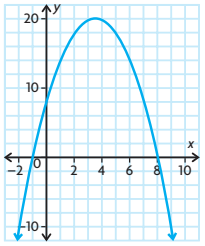
1. **a)** Yes
b) No; it contains a rational exponent.
c) Yes
d) No; it is a rational function.

2. **a)** Answers may vary. For example, $f(x) = x^3 + 2x^2 - 8x + 1$.
b) Answers may vary. For example, $f(x) = 5x^4 - x^2 - 7$.
c) Answers may vary. For example, $f(x) = 7x^6 + 3$.
d) Answers may vary. For example, $f(x) = -2x^5 - 4x^4 + 3x^3 - 2x^2 + 9$.

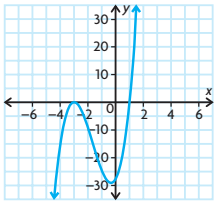
3. **a)** As $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to -\infty$.
b) As $x \to \pm\infty$, $y \to \infty$.
c) As $x \to -\infty$, $y \to -\infty$ and as $x \to \infty$, $y \to \infty$.
d) As $x \to \pm\infty$, $y \to -\infty$.

4. **a)** even **c)** odd
b) odd **d)** even

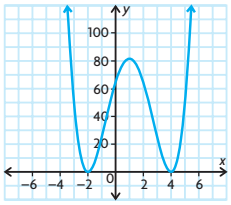
5. Answers may vary. For example:
a)



b)



c)



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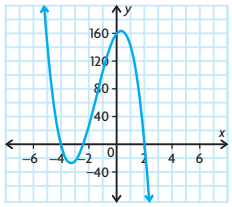


6. end behaviours

7. $y = 5(x - 2)(x + 3)^2(x - 5)$

8. **a)** reflection in the x-axis, vertical stretch by a factor of 25, horizontal compression by a factor of $\frac{1}{3}$, horizontal translation 4 units to the left, vertical translation 60 units down
b) vertical stretch by a factor of 8, horizontal stretch by a factor of $\frac{4}{3}$, vertical translation 43 units up
c) reflection in the y-axis, horizontal compression by a factor of $\frac{1}{13}$, horizontal translation 2 units to the right, vertical translation 13 units up
d) vertical compression by a factor of $\frac{8}{11}$, reflection in the y-axis, vertical translation 1 unit down

9. vertically stretched by a factor of 5, horizontally translated 4 units to the left, and vertically translated 2 units down

Lesson 3.5, pp. 168–170

1. **a)** **i)** $x^3 - 14x^2 - 24x - 38$ remainder -87
ii) $x^3 - 20x^2 + 84x - 326$ remainder 1293
iii) $x^3 - 15x^2 - 11x - 1$ remainder -12
b) No; because for each division problem there is a remainder.

2. **a)** 2 **b)** 2 **c)** 1 **d)** not possible

3. **a)** $x^2 - 15x + 6$ remainder $-48x + 14$
b) $5x^2 - 19x + 60$ remainder -184
c) $x - 6$ remainder $-6x^2 + 22x + 6$
d) Not possible

4.

Dividend	Divisor	Quotient	Remainder
$2x^3 - 5x^2 + 8x + 4$	$x + 3$	$2x^2 - 11x + 41$	-119
$6x^4 + 12x^3 - 10x^2 - 4x + 29$	$2x + 4$	$3x^3 - 5x + 8$	-3
$6x^4 + 2x^3 + 3x^2 - 11x - 9$	$3x + 1$	$2x^3 + x - 4$	-5
$3x^3 + x^2 - 6x + 16$	$x + 2$	$3x^2 - 5x + 4$	8

5. **a)** $x^2 + 4x + 14$ remainder 57
b) $x^2 - 6$ remainder 13
c) $x^2 + 2x - 3$ remainder -2
d) $x^2 + 3x - 9$ remainder $-16x + 62$
e) $x + 1$ remainder $8x^2 - 8x + 11$
f) $x + 3$ remainder $-4x^3 - 4x^2 + 8x + 14$

6. **a)** $x^2 + 3x + 2$ no remainder
b) $2x^2 - 5x - 12$ remainder 7
c) $6x^3 - 5x^2 - 19x + 10$ remainder -2
d) $x^2 + 2x - 8$ remainder -2
e) $6x^3 - 31x^2 + 45x - 18$ no remainder
f) $3x^2 - 1$ no remainder

7. **a)** $x^3 + 4x^2 - 51x + 89$
b) $3x^4 - 2x^3 + 3x^2 - 38x + 39$
c) $5x^4 + 22x^3 - 17x^2 + 21x + 10$
d) $x^6 + 8x^5 + 5x^4 - 13x^3 - 72x^2 + 49x - 3$

8. **a)** $r = 20$ **c)** $r = 0$
b) $r = x - 22$ **d)** $r = 2x^2 + 2$

9. **a)** $x + 3$ **c)** $x + 4$
b) $x + 10$ **d)** $x - 2$

10. **a)** $x + 5$ is a factor since there is no remainder.
b) $x + 2$ is a factor since there is no remainder.
c) $x - 2$ is not a factor since there is a remainder of 2.
d) $x - 1$ is not a factor since there is a remainder of 1.
e) $3x + 5$ is not a factor since there is a remainder of $-\frac{13}{3}$.
f) $5x - 1$ is not a factor since there is a remainder of -8.

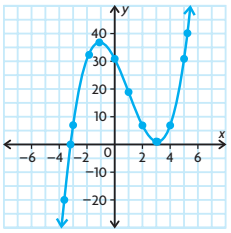
11. $(x + 1)$ cm

12. **a)** 7 **b)** 3

13. 2

14. Yes, $f(x)$ is always divisible by $x - 1$. Regardless of the value of n, $f(x) = x^n - 1$ can always be written as $f(x) = x^n + 0x^{n-1} + 0x^{n-2} + \ldots 0x - 1$. Therefore, the same pattern continues when dividing $x^n - 1$ by $x - 1$, regardless of how large n is, and there is never a remainder.

15. **a)** $f(x) = (x^3 - 3x^2 - 10x + 31) = (x - 4)(x^2 + x - 6)$ remainder 7
b) $f(x) = (x^3 - 3x^2 - 10x + 31) = (x - 4)(x + 3)(x - 2)$ remainder 7
c)



16. Answers may vary. For example:

$$x - 3 \overline{)2x^4 + 3x^3 - 25x^2 - 7x - 14}$$

$$2x^3 = 2x^3 + 9x^2 + 2x - 1$$

$2x^3(x - 3) \rightarrow \quad 2x^4 - 6x^3$

$9x^3 - 25x^2$

$9x^2(x - 3) \rightarrow \quad 9x^3 - 27x^2$

$2x^2 - 7x$

$2x(x - 3) \rightarrow \quad 2x^2 - 6x$

$-1x - 14$

$-1(x - 3) \rightarrow \quad -1x + 3$

-17

17. $r = 2x + 5$ cm

18. **a)** $x^2 + xy + y^2$
b) $x^2 - 2xy + y^2$

19. $x - y$ is a factor because there is no remainder.

20. $[q(x) + 1](x + 5)$

Lesson 3.6, pp. 176–177

1. **a)** **i)** 64
ii) 22
iii) 12
b) No, according to the factor theorem, $x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.

2. **a)** not divisible by $x - 1$
b) divisible by $x - 1$
c) not divisible by $x - 1$
d) divisible $x - 1$

3. $(x + 1)(x + 3)(x - 2)$

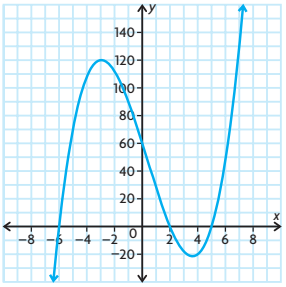
4. **a)** -1 **c)** 0 **e)** 30
b) -5 **d)** -34 **f)** 0

5. **a)** yes **c)** yes
b) no **d)** no

6. **a)** $(x - 2)(x - 4)(x + 3)$
b) $(x - 1)(2x + 3)(2x + 5)$
c) $x(x - 2)(x + 4)(x + 6)$
d) $(x + 2)(x + 5)(4x - 9)(x - 3)$
e) $x(x + 2)(x + 1)(x - 3)(x - 5)$
f) $(x - 3)(x - 3)(x + 4)(x + 4)$

7. **a)** $(x - 2)(x + 5)(x + 6)$
b) $(x + 1)(x - 3)(x + 2)$
c) $(x + 1)(x - 1)(x - 2)(x + 2)$
d) $(x - 2)(x + 1)(x + 8)(x - 4)$
e) $(x - 1)(x^2 + 1)$
f) $(x - 1)(x^2 + 1)(x^2 + 1)$

8. **a)**



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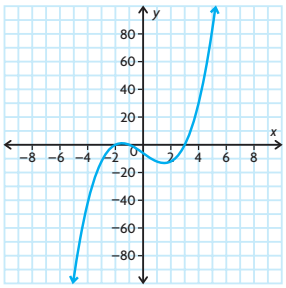

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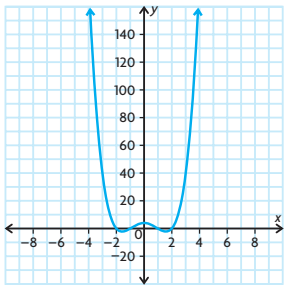
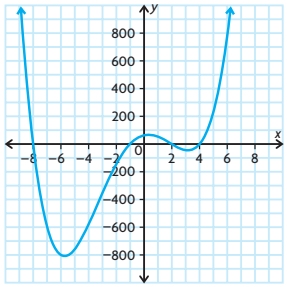
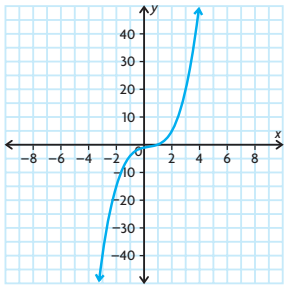


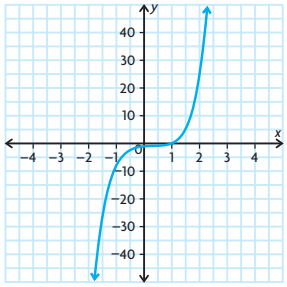
d)



e)



f)



9. 20

10. $a = 6, b = 3$

11. For $x^n - a^n$, if n is even, they're both factors. If n is odd, only $(x - a)$ is a factor. For $x^n + a^n$, if n is even, neither is a factor. If n is odd, only $(x + a)$ is a factor.

12. $a = -2, b = 22$;
The other factor is $-2x + 3$.

13. -6

14. $x^4 - a^4$
$= (x^2)^2 - (a^2)^2$
$= (x^2 + a^2)(x^2 - a^2)$
$= (x^2 + a^2)(x + a)(x - a)$

15. Answers may vary. For example: if $f(x) = k(x - a)$, then $f(a) = k(a - a) = k(0) = 0$.

16. $x^2 - x - 2 = (x - 2)(x + 1)$;
If $f(x) = x^3 - 6x^2 + 3x + 10$, then $f(2) = 0$ and $f(-1) = 0$.

17. If $f(x) = (x + a)^5 + (x + c)^5 + (a - c)^5$, then $f(-a) = 0$

Lesson 3.7, p. 182

1. $(x + b)(x^2 - bx + b^2)$

2. **a)** $(x - 4)(x^2 + 4x + 16)$
b) $(x - 5)(x^2 + 5x + 25)$
c) $(x + 2)(x^2 - 2x + 4)$
d) $(2x - 3)(4x^2 + 6x + 9)$
e) $(4x - 5)(16x^2 + 20x + 25)$
f) $(x + 1)(x^2 - x + 1)$
g) $(3x + 2)(9x^2 - 6x + 4)$
h) $(10x + 9)(100x^2 - 90x + 81)$
i) $8(3x - 1)(9x^2 + 3x + 1)$

3. **a)** $(4x + 3y)(16x^2 - 12xy + 9y^2)$
b) $(-3x)(x - 2)(x^2 + 2x + 4)$
c) $(4 - x)(7x^2 + 25x + 31)$
d) $(x^2 + 4)(x^4 - 4x^2 + 16)$

4. **a)** $(x - 7)(x^2 + 7x + 49)$
b) $(6x - 1)(36x^2 + 6x + 1)$
c) $(x + 10)(x^2 - 10x + 100)$
d) $(5x - 8)(25x^2 + 40x + 64)$
e) $(4x - 11)(16x^2 + 44x + 121)$
f) $(7x + 3)(49x^2 - 21x + 9)$
g) $(8x + 1)(64x^2 - 8x + 1)$
h) $(11x + 12)(121x^2 - 132x + 144)$
i) $(8 - 11x)(64 + 88x + 121x^2)$

5. **a)** $\left(\frac{1}{3}x - \frac{2}{5}\right)\left(\frac{1}{9}x^2 + \frac{2}{15}x + \frac{4}{25}\right)$
b) $-16x^2(3x + 2)(9x^2 - 6x + 4)$
c) $7(4x - 5)(x^2 - x + 1)$
d) $\left(\frac{1}{2}x - 2\right)\left(\frac{1}{4}x^2 + x + 4\right)\left(\frac{1}{64}x^6 + x^3 + 64\right)$

6. Agree; by the formulas for factoring the sum and difference of cubes, the numerator of the fraction is equivalent to $(a^3 + b^3) + (a^3 - b^3)$. Since $(a^3 + b^3) + (a^3 - b^3) = 2a^3$, the entire fraction is equal to 1.

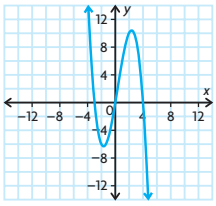
7. **a)** $1^3 + 12^3 = (1 + 12)(1^2 - (1)(12) + 12^2)$
$= (13)(133) = 1729$
b) $9^3 + 10^3 = (9 + 10)(9^2 - (9)(10) + 10^2)$
$= (19)(91) = 1729$

8. $x^9 + y^9$
$= x^{18} + 2x^9y^9 + y^{18}$
$= (x^{18} + y^{18}) + 2x^9y^9$
$= (x^6 + y^6)(x^{12} - x^6y^6 + y^{12}) + 2x^9y^9$
$= (x^2 + y^2)(x^4 - x^2y^2 + y^4)(x^{12} - x^6y^6 + y^{12}) + 2x^9y^9$

9. Answers may vary. For example, this statement is true because $a^3 - b^3$ is the same as $a^3 + (-b)^3$.

10. **a)** 1729 was the number of the taxicab that G. H. Hardy rode in when going to visit the mathematician Ramanujan. When Hardy told Ramanujan that the number of the taxicab he rode in was uninteresting, Ramanujan replied that the number was interesting because it was the smallest number that could be expressed as the sum of two cubes in two different ways. This is how such numbers came to be known as taxicab numbers.
b) Yes;
$TN(1) = 2$
$TN(2) = 1729$
$TN(3) = 87\,539\,319$
$TN(4) = 6\,963\,472\,309\,248$
$TN(5) = 48\,988\,659\,276\,962\,496$
$TN(6) = 24\,153\,319\,581\,254\,312\,065\,344$

Chapter Review, pp. 184–185

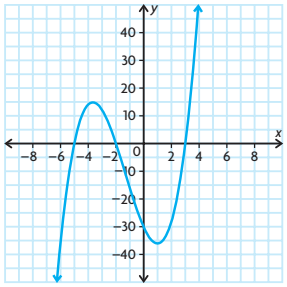
1.


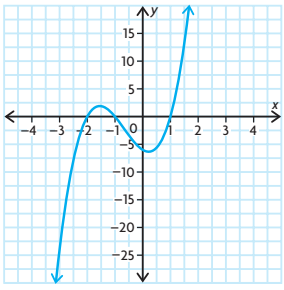
2. As $x \to -\infty, y \to +\infty$ and as $x \to \infty, y \to \infty$.

3. **a)** degree: 2 + 1; leading coefficient: positive; turning points: 2
b) degree: 3 + 1; leading coefficient: positive; turning points: 3

4. **a)** Answers may vary. For example,
$f(x) = (x + 3)(x - 6)(x - 4)$,
$f(x) = 10(x + 3)(x - 6)(x - 4)$,
$f(x) = -4(x + 3)(x - 6)(x - 4)$
b) Answers may vary. For example,
$f(x) = (x - 5)(x + 1)(x + 2)$,
$f(x) = -6(x - 5)(x + 1)(x + 2)$,
$f(x) = 9(x - 5)(x + 1)(x + 2)$
c) Answers may vary. For example,
$f(x) = (x + 7)(x - 2)(x - 3)$,
$f(x) = \frac{1}{4}(x + 7)(x - 2)(x - 3)$,
$f(x) = 3(x + 7)(x - 2)(x - 3)$
d) Answers may vary. For example,
$f(x) = (x - 9)(x + 5)(x + 4)$,
$f(x) = 7(x - 9)(x + 5)(x + 4)$,
$f(x) = -\frac{1}{3}(x - 9)(x + 5)(x + 4)$

5. **a)** Answers may vary. For example,
$f(x) = (x + 6)(x - 2)(x - 5)(x - 8)$,
$f(x) = 2(x + 6)(x - 2)(x - 5)(x - 8)$,
$f(x) = -8(x + 6)(x - 2)(x - 5)(x - 8)$
b) Answers may vary. For example,
$f(x) = (x - 4)(x + 8)(x - 1)(x - 2)$,
$f(x) = \frac{3}{4}(x - 4)(x + 8)(x - 1)(x - 2)$,
$f(x) = -12(x - 4)(x + 8)(x - 1)(x - 2)$
c) Answers may vary. For example,
$f(x) = x(x + 1)(x - 9)(x - 10)$,
$f(x) = 5x(x + 1)(x - 9)(x - 10)$,
$f(x) = -3x(x + 1)(x - 9)(x - 10)$
d) Answers may vary. For example,
$f(x) = (x + 3)(x - 3)(x + 6)(x - 6)$,
$f(x) = \frac{2}{5}(x + 3)(x - 3)(x + 6)(x - 6)$,
$f(x) = -10(x + 3)(x - 3)(x + 6)(x - 6)$

6.


7. $y = 3(x - 1)(x + 1)(x + 2)$


8. **a)** reflected in the x-axis, vertically stretched by a factor of 2, horizontally translated 1 unit to the right, and vertically translated 23 units up
b) horizontally stretched by a factor of $\frac{13}{12}$, horizontally translated 9 units to the left, and vertically translated 14 units down
c) horizontally translated 4 units to the right
d) horizontally translated $\frac{3}{7}$ units to the left
e) vertically stretched by a factor of 40, reflected in the y-axis, horizontally compressed by a factor of $\frac{1}{7}$, horizontally translated 10 units to the right, and vertically translated 9 units up

9. **a)** Answers will vary. For example, $(-2, -5400)$, $(3, 0)$, and $(8, 5400)$.
b) Answers will vary. For example, $(-7, -18)$, $(0, -19)$, and $(7, -20)$.
c) Answers will vary. For example, $\left(-6, \frac{182}{11}\right)$, $(-5, 16)$, and $\left(-4, \frac{170}{11}\right)$.
d) Answers will vary. For example, $(-2, -86)$, $(0, 14)$, and $(2, 114)$.
e) Answers will vary. For example, $(-1, -44)$, $(0, -45)$, and $(1, -46)$.
f) Answers will vary. For example, $(5, 1006)$, $(12, 6)$, and $(19, -994)$.

10. **a)** $2x^2 - 5x + 28$ remainder -144
b) $x^2 + 4x + 5$ remainder $26x + 33$
c) $2x - 6$ remainder $10x^2 + 27x - 34$
d) $x - 4$ remainder $4x^3 + 17x^2 - 8x - 18$

11. **a)** $(x + 2)(2x^2 + x - 3)$ remainder 1
b) $(x + 2)(3x^2 + 7x + 3)$ remainder -3
c) $(x + 2)(2x^3 + x^2 - 18x - 9)$ remainder 0
d) $(x + 2)(2x^2 - 5)$ remainder 6

12. **a)** $2x^3 - 7x^2 - 107x + 75$
b) $4x^4 + 3x^3 - 8x^2 + 22x + 17$
c) $3x^4 + 14x^3 - 42x^2 + 3x + 33$
d) $3x^6 - 11x^5 - 9x^4 + 47x^3 - 46x + 14$

13. 13

14. **a)** $(x + 1)(x - 8)(x + 2)$
b) $(x - 4)(2x + 3)(x + 3)$
c) $x(x - 2)(x - 3)(3x - 4)$
d) $(x - 1)(x + 4)(x + 4)(x + 4)$

15. **a)** $(x - 2)(4x + 5)(2x - 1)$
b) $(2x + 5)(x - 2)(x + 3)$
c) $(x - 3)(x - 3)(x - 3)(x + 2)$
d) $(2x + 1)(2x + 1)(x - 3)(x + 3)$

16. **a)** $(4x - 3)(16x^2 + 12x + 9)$
b) $(8x - 5)(64x^2 + 40x + 25)$
c) $(7x - 12)(49x^2 + 84x + 144)$
d) $(11x - 1)(121x^2 + 11x + 1)$

17. **a)** $(10x + 7)(100x^2 - 70x + 49)$
b) $(12x + 5)(144x^2 - 60x + 25)$
c) $(3x + 11)(9x^2 - 33x + 121)$
d) $(6x + 13)(36x^2 - 78x + 169)$

18. **a)** $(x - y)(x^2 + xy + y^2)(x + y)(x^2 - xy + y^2)$

b) $(x - y)(x + y)(x^4 + x^2y^2 + y^4)$

c) Both methods produce factors of $(x - y)$ and $(x + y)$; however, the other factors are different. Since the two factorizations must be equal to each other, this means that $(x^4 + x^2y^2 + y^4)$ must be equal to $(x^2 + xy + y^2)(x^2 - xy + y^2)$.

Chapter Self-Test, p. 186

1. **a)** $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number. The degree of the function is n; the leading coefficient is a_n.

b) $n - 1$

c) n

d) odd degree function

e) even degree function with a negative leading coefficient

2. $y = (x + 4)(x + 2)(x - 2)$

3. **a)** $(x - 9)(x + 8)(2x - 1)$

b) $(3x - 4)(3x^2 + 9x + 79)$

4. more zeros

5. $-5 < x < -3; x > 1$

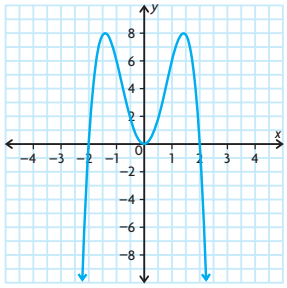
6. yes

7. **a)** $y = 5(2(x - 2))^3 + 4$

b) (2.5, 9)

8. $x + 5$

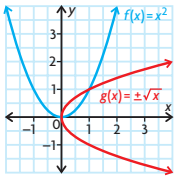
9. $a = -2$; zeros at 0, -2, and 2.



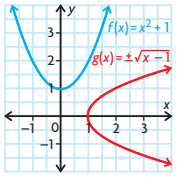
Cumulative Review Chapters 1–3, pp. 188–191

1.	(b)	**9.**	(c)	**17.**	(a)	**25.**	(c)
2.	(a)	**10.**	(d)	**18.**	(d)	**26.**	(c)
3.	(c)	**11.**	(a)	**19.**	(b)	**27.**	(d)
4.	(b)	**12.**	(a)	**20.**	(c)	**28.**	(b)
5.	(b)	**13.**	(c)	**21.**	(b)	**29.**	(c)
6.	(d)	**14.**	(d)	**22.**	(b)	**30.**	(c)
7.	(d)	**15.**	(c)	**23.**	(b)	**31.**	(c)
8.	(a)	**16.**	(c)	**24.**	(a)		

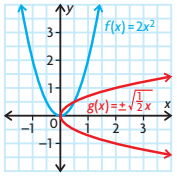
32. **a)**



b) Answers may vary. For example, vertical translation up produces horizontal translation of the inverse to the right.



Vertical stretch produces horizontal stretch of inverse.



c) Answers may vary. For example, if the vertex of the inverse is (a, b), restrict the value of y to either $y \geq b$ or $y \leq b$.

33. Answers may vary. For example, average rates of change vary between -2 and 4, depending on the interval; instantaneous rates of change are 9 at $(0, 1)$, 0 at $(1, 5)$, -3 at $(2, 3)$, 0 at $(3, 1)$, 9 at $(4, 5)$; instantaneous rate of change is 0 at maximum $(1, 5)$ and at minimum $(3, 1)$.

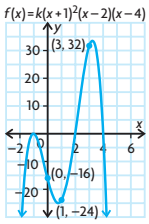
34. **a)** $f(x) = -2(x + 1)^2(x - 2)(x - 4)$

b) $p = 32$

c) As $x \to \pm\infty, f(x) \to -\infty$; zeros: -1, 2, and 4

d) -16

e)



Chapter 4

Getting Started, pp. 194–195

1. **a)** 3 **c)** 1

b) 5 **d)** $\frac{64}{11}$

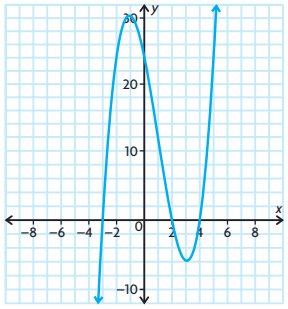
2. **a)** $x(x + 6)(x - 5)$

b) $(x - 4)(x^2 + 4x + 16)$

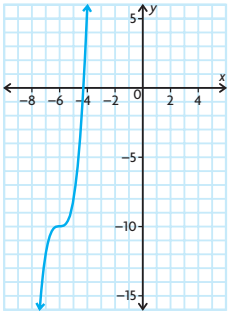
c) $3x(2x + 3)(4x^2 - 6x + 9)$

d) $(x + 3)(x - 3)(2x + 7)$

3. **a)**



b)



4. 2 and 5

5. **a)** 3 and -3 **c)** $-\frac{2}{3}$ and $\frac{5}{2}$

b) -10 and 2 **d)** 0.3452 and -4.345

6. **a)** (3, 7); Answers may vary. For example, the change in distance over time from $t = 3$ to $t = 7$ is greater than at other intervals of time.

b) $\frac{1}{3}$ m/s; $\frac{3}{4}$ m/s

c) Answers may vary. For example, away; Erika's displacement, or distance from the sensor, is increasing.

7. **a)** 2 s

b) 4.75 m/s

c) -10.245 m/s

8. **a)** Disagree; You could use the quadratic formula to solve $y = x^3 + 4x^2 + 3x$ because it equals $x(x^2 + 4x + 3)$.

b) Disagree; $y = (x + 3)^2(x - 2)$ is a cubic equation that will have two roots.

c) Disagree; The equation $y = x^3$ will only pass through two quadrants.

d) Agree; All polynomials are continuous and all polynomials have a y-intercept.

e) Disagree; $f(-3) = 9$

f) Agree; The instantaneous rates of change will tell you whether the graph is increasing, decreasing, or not changing at those points.

Lesson 4.1, pp. 204–206

1. **a)** $0, 1, -2, 2$ **d)** $-6, \frac{5}{2}$

b) $-\frac{3}{2}, \frac{5}{4}, -7$ **e)** $0, -3, 3$

c) $3, -5, 4$ **f)** $-5, -2, 6$

b) $(x - y)(x + y)(x^4 + x^2y^2 + y^4)$

c) Both methods produce factors of $(x - y)$ and $(x + y)$; however, the other factors are different. Since the two factorizations must be equal to each other, this means that $(x^4 + x^2y^2 + y^4)$ must be equal to $(x^2 + xy + y^2)(x^2 - xy + y^2)$.

Chapter Self-Test, p. 186

1. **a)** $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$, where $a_0, a_1, \ldots, a_n$ are real numbers and n is a whole number. The degree of the function is n; the leading coefficient is a_n.

b) $n - 1$

c) n

d) odd degree function

e) even degree function with a negative leading coefficient

2. $y = (x + 4)(x + 2)(x - 2)$

3. **a)** $(x - 9)(x + 8)(2x - 1)$

b) $(3x - 4)(3x^2 + 9x + 79)$

4. more zeros

5. $-5 < x < -3; x > 1$

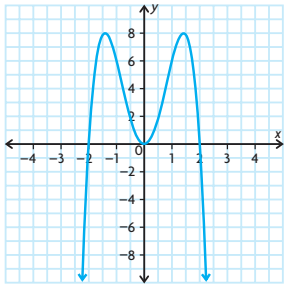
6. yes

7. **a)** $y = 5(2(x - 2))^3 + 4$

b) (2.5, 9)

8. $x + 5$

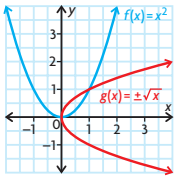
9. $a = -2$; zeros at 0, -2, and 2.



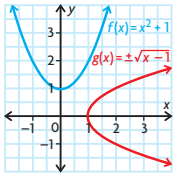
Cumulative Review Chapters 1–3, pp. 188–191

1.	(b)	**9.**	(c)	**17.**	(a)	**25.**	(c)
2.	(a)	**10.**	(d)	**18.**	(d)	**26.**	(c)
3.	(c)	**11.**	(a)	**19.**	(b)	**27.**	(d)
4.	(b)	**12.**	(a)	**20.**	(c)	**28.**	(b)
5.	(b)	**13.**	(c)	**21.**	(b)	**29.**	(c)
6.	(d)	**14.**	(d)	**22.**	(b)	**30.**	(c)
7.	(d)	**15.**	(c)	**23.**	(b)	**31.**	(c)
8.	(a)	**16.**	(c)	**24.**	(a)		

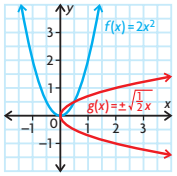
32. **a)**



b) Answers may vary. For example, vertical translation up produces horizontal translation of the inverse to the right.



Vertical stretch produces horizontal stretch of inverse.



c) Answers may vary. For example, if the vertex of the inverse is (a, b), restrict the value of y to either $y \geq b$ or $y \leq b$.

33. Answers may vary. For example, average rates of change vary between -2 and 4, depending on the interval; instantaneous rates of change are 9 at $(0, 1)$, 0 at $(1, 5)$, -3 at $(2, 3)$, 0 at $(3, 1)$, 9 at $(4, 5)$; instantaneous rate of change is 0 at maximum $(1, 5)$ and at minimum $(3, 1)$.

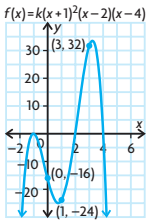
34. **a)** $f(x) = -2(x + 1)^2(x - 2)(x - 4)$

b) $p = 32$

c) As $x \to \pm\infty, f(x) \to -\infty$; zeros: -1, 2, and 4

d) -16

e)



Chapter 4

Getting Started, pp. 194–195

1. **a)** 3 **c)** 1

b) 5 **d)** $\frac{64}{11}$

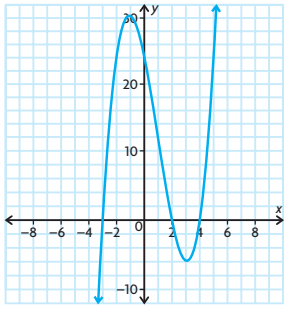
2. **a)** $x(x + 6)(x - 5)$

b) $(x - 4)(x^2 + 4x + 16)$

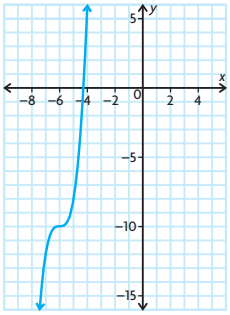
c) $3x(2x + 3)(4x^2 - 6x + 9)$

d) $(x + 3)(x - 3)(2x + 7)$

3. **a)**



b)



4. 2 and 5

5. **a)** 3 and -3 **c)** $-\frac{2}{3}$ and $\frac{5}{2}$

b) -10 and 2 **d)** 0.3452 and -4.345

6. **a)** (3, 7); Answers may vary. For example, the change in distance over time from $t = 3$ to $t = 7$ is greater than at other intervals of time.

b) $\frac{1}{3}$ m/s; $\frac{3}{4}$ m/s

c) Answers may vary. For example, away; Erika's displacement, or distance from the sensor, is increasing.

7. **a)** 2 s

b) 4.75 m/s

c) -10.245 m/s

8. **a)** Disagree; You could use the quadratic formula to solve $y = x^3 + 4x^2 + 3x$ because it equals $x(x^2 + 4x + 3)$.

b) Disagree; $y = (x + 3)^2(x - 2)$ is a cubic equation that will have two roots.

c) Disagree; The equation $y = x^3$ will only pass through two quadrants.

d) Agree; All polynomials are continuous and all polynomials have a y-intercept.

e) Disagree; $f(-3) = 9$

f) Agree; The instantaneous rates of change will tell you whether the graph is increasing, decreasing, or not changing at those points.

Lesson 4.1, pp. 204–206

1. **a)** $0, 1, -2, 2$ **d)** $-6, \frac{5}{2}$

b) $-\frac{3}{2}, \frac{5}{4}, -7$ **e)** $0, -3, 3$

c) $3, -5, 4$ **f)** $-5, -2, 6$

2. **a)** $0, -3, 3$

b) ± 3

c) $0, 2, -2, -\frac{5}{3}$

d) $0, \frac{2}{5}, 3$

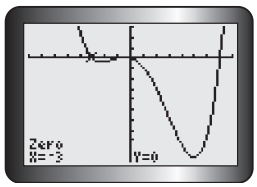
e) $-3\sqrt[3]{3}$

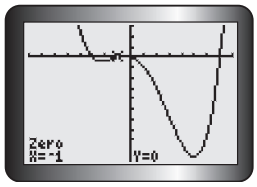
f) $0, \pm 2\sqrt{6}$

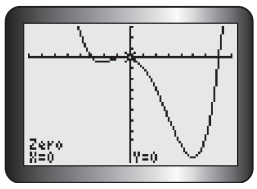
3. **a)** $6, -1, \frac{7}{2}$

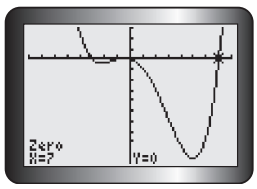
b) $2x^3 - 17x^2 + 23x + 42 = 0$ or $(x - 6)(x + 1)(2x - 7)$

4. Algebraically:
$x = -1, -3, 7, 0$
Graphically:

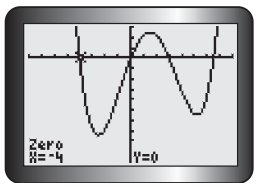


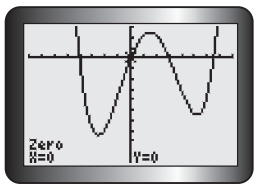


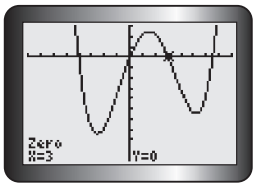


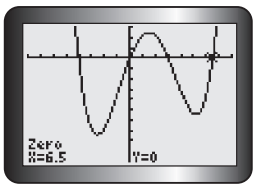


5. $0, 3, -4, \frac{13}{2}$









6. **a)** $0, 2, -5$

b) $-1, 17$

c) 2

d) $0, -2, -5, 5$

e) $0, -3, 4$

f) -1

7. **a)** $-3, 6, 5$

b) $1, -2, -3, -5$

c) $-1, \frac{1}{2}, 3$

d) $-1, \frac{3}{2}, -2$

e) $2, -4, \frac{1}{2}, \frac{5}{2}$

f) $\frac{1}{2}, \frac{5}{3}, \frac{3}{2}$

8. **a)** $-3, 1, 2$

b) $-2, -1.24, 1, 7.24$

c) $-2, 1$

d) $-3, 0, 2$

e) $-0.86, 1.8, 2.33$

f) $-2.71, -0.16$

9. **a)** $3, -2, 5$

b) $0, 2, \frac{4}{3}$

c) $2, -2, -\frac{1}{3}, \frac{5}{2}$

d) $0, 3$

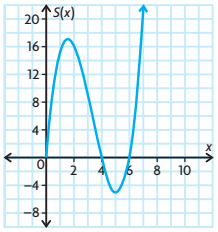
10. 3, 4.92; either 3 cm by 3 cm or 4.92 cm by 4.92 cm.

11. **a)** 4 and 6

b) 5

c) 2

d)



This is not a good model to represent Maya's score because the graph is shown for real numbers, but the number of games can only be a whole number.

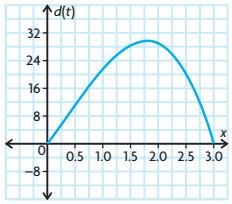
12. 22.59 s

13. **a)** $d(t) = -3t(t + 2)(t - 3)$

b) 3 h after departure

c) -2, because time cannot be negative

d)



e) 1.8 h after departure

14. **a)** $0 \leq t \leq 5$

b) Answers may vary. For example, because the function involves decimals, graphing technology would be the better strategy for answering the question.

c) 0.25 L

15. All powers are even, which means every term is positive for all real numbers. Thus, the polynomial is always positive.

16. For $x = 1$, the left side is -48.
For $x = -1$, the left side is -12.

17. **a)** Answers may vary. For example, $x^3 + x^2 - x - 1 = 0$; $F(1) = 0$, so it is simple to solve using the factor theorem.

b) Answers may vary. For example, $x^2 - 2x = 0$; The common factor, x, can be factored out to solve the equation.

c) Answers may vary. For example, $x^3 - 2x^2 - 9x + 18$; An x can be factored out of the first two terms and a -2 out of the second two terms leaving you with the factors $(x - 2)(x^2 - 9)$.

d) Answers may vary. For example, $10x^2 - 7x + 1 = 0$; The roots are fractional, which makes using the quadratic formula the most sensible approach.

e) $x^3 - 8 = 0$; This is the difference of two cubes.

f) $0.856x^3 - 2.74x^2 + 0.125x - 2.89 = 0$; The presence of decimals makes using graphing technology the most sensible strategy.

18. **a)** $0 = x^4 + 10$. x^4 is non-negative for all real x, so $x^4 + 10$ is always positive.

b) A degree 5 polynomial function $y = f(x)$ has opposite end behaviour, so somewhere in the middle it must cross the x-axis. This means its corresponding equation $0 = f(x)$ will have at least one real root.

19. $y = x^5 + x + 1$; By the factor theorem, the only possible rational zeros are 1 and -1. Neither works. Because the degree is odd, the polynomial has opposite end behaviour, and hence must have at least one zero, which must be irrational.

Lesson 4.2, pp. 213–215

1. **a)** $x \leq 4$; $\{x \in \mathbf{R} \mid x \leq 4\}$
b) $x < 7$; $\{x \in \mathbf{R} \mid x < 7\}$
c) $x < -5$; $\{x \in \mathbf{R} \mid x < 5\}$
d) $x \geq -3$; $\{x \in \mathbf{R} \mid x \geq -3\}$
e) $x > -10$; $\{x \in \mathbf{R} \mid x > -10\}$
f) $x \geq 7$; $\{x \in \mathbf{R} \mid x \geq 7\}$

2. **a)** $x \in [-3, \infty)$
b) $x \in \left(-\infty, -\frac{2}{3}\right)$
c) $x \in [18, \infty)$
d) $x \in [1, \infty)$
e) $x \in (-\infty, 0)$
f) $x \in [-10, \infty)$

3. $-1 \leq x < 6$



4. **a)** yes **b)** no **c)** no **d)** no **e)** yes **f)** no

5. **a)** $x \leq 7$ **b)** $x < 0$ **c)** $x < -10$ **d)** $x \geq 5$ **e)** $x < 6$ **f)** $x \geq \frac{7}{5}$

6. **a)** yes **b)** yes **c)** no **d)** no **e)** yes **f)** no

7. **a)** $-6 < x < 2$
b) $4 < x < 8$
c) $-4 \leq x \leq 10$
d) $-7 \leq x \leq -4$
e) $7 < x < 9$
f) $-3 \leq x \leq -\frac{1}{2}$

8. **a)** Answers may vary. For example, $3x + 1 > 9 + x$
b) Answers may vary. For example, $3x + 1 \leq 4 + x$

9. **a)** $\{x \in \mathbf{R} \mid -6 \leq x \leq 4\}$
b) $-13 \leq 2x - 1 \leq 7$

10. Attempting to solve $x - 3 < 3 - x < x - 5$ yields $3 > x > 4$, which has no solution. Solving $x - 3 > 3 - x > x - 5$ yields $3 < x < 4$.

11. **a)** $\frac{1}{2}x + 1 < 3$
b) $x < 4$
c) $\frac{1}{2}x + 1 < 3$

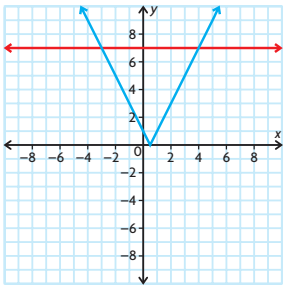
$$\frac{1}{2}x < 2$$
$$x < 4$$

12. **a)** $18 \leq \frac{5}{9}(F - 32) \leq 22$
b) $64.4 \leq F \leq 71.6$

13. 18 min

14. **a)** $\frac{9}{5}C + 32 = F$
b) $C > -40$

15. **a)**



b) $-3 < x < 4$

16. The solution will always have an upper and lower bound due to the manner in which the inequality is solved. The only exception to this is when there is no solution set.

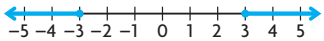
17. **a)** Isolating x is very hard.
b) A graphical approach as described in the lesson yields a solution of $x > 2.75$ (rounded to two places).

18. **a)** Maintained
b) Maintained if both positive; switched if both negative; varies if one positive and one negative.
c) Maintained
d) Switched
e) Switched unless one is positive and the other is negative, in which case it is maintained. (If either side is zero, it becomes undefined.)
f) Maintained, except that $<$ and $>$ become $\leq$ and $\geq$, respectively.
g) Maintained, but it is undefined for negative numbers.

19. **a)** $\{x \in \mathbf{R} \mid -2 < x < 2\}$; $(-2, 2)$



b) $\{x \in \mathbf{R} \mid -3 \leq x \geq 3\}$; $(-\infty, -3)$ or $(3, \infty)$



c) $\{x \in \mathbf{R} \mid -5 < x < 3\}$; $(-5, 3)$



d) $\{x \in \mathbf{R} \mid x \leq 3\}$; $(-\infty, 3)$



Mid-Chapter Review, p. 218

1. **a)** $0, \frac{5}{2}, 4$ **b)** -2 **c)** $1, -2, 5$ **d)** $-4, 6, 5, -5$ **e)** $0, -2, -9$ **f)** $3, -3, 2, -2$

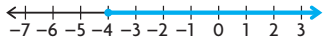
2. **a)** $h(t) = -5t^2 + 3t + 24.55$
b) 24.55 m
c) 2.5 s after jumping
d) $t > 2.5$ s; Jude is below sea level (in the water)

3. either 10 cm by 10 cm or 1.34 cm by 1.34 cm

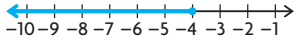
4. **a)** $x > -11$



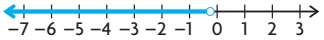
b) $x \geq -4$



c) $x \leq -4$



d) $x < -\frac{1}{3}$



5. $x \in [-2, 6)$

6. **a)** Answers may vary. For example, $2x + 1 > 15$
b) Answers may vary. For example, $4x - 1 < -33$
c) Answers may vary. For example, $-3 \leq 2x - 1 \leq 13$
d) Answers may vary. For example, $x - 2 \leq 3x - 8$

7. **a)** $f(x) = -x + 1$; $g(x) = 2x - 5$
b) $x > 2$
c)
$$f(x) < g(x)$$
$$-x + 1 < 2x - 5$$
$$-3x < -6$$
$$x > 2$$

8. **a)** $N(t) = 20 + 0.02t$; $M(t) = 15 + 0.03t$
b) $20 + 0.02t > 15 + 0.03t$
c) $0 \leq t < 500$
d) Negative time has no meaning.

Lesson 4.3, pp. 225–228

1. **a)** $-2 \leq x \leq -1$ or $x \geq 3$
b) $-3 < x < 2$ or $x > 4$
c) $x < -\frac{2}{5}$ or $\frac{3}{4} < x < 3$
d) $-\frac{1}{4} \leq x \leq \frac{5}{2}$ or $x \geq 5$

2. **a)** $(-\infty, -5]$, $[-2, 0]$, and $[3, \infty)$
b) $x = 1$
c) $[-7, -3]$ and $[0, 4]$
d) $(-\infty, -4]$ and $[2, 7]$

3. $-1 < x < 2$ or $x > 3$

4. $-1.14 < x < 3$ and $x > 6.14$

5. **a)** $(-1, 2)$, $(4, \infty)$
b) $(-2, 2)$, $(2, \infty)$
c) $(-\infty, -2)$, $(0, 1)$
d) $(-\infty, 2)$, $(2, \infty)$

6. **a)** $x < -1$ or $x > 1$
b) $-3 < x < 4$
c) $x \leq -\frac{1}{2}$ or $x \geq 5$
d) $-7 < x < 0$ or $x > 2$
e) $-\frac{3}{2} < x < 3$ or $x > 3$
f) $-4 \leq x \leq \frac{3}{2}$

Answers

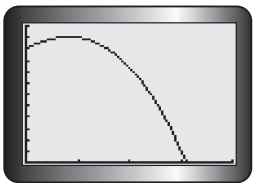
7. **a)** $x \leq -1$ or $x \geq 7$
b) $0 < x < 2$
c) $x \leq -3$ or $-2 \leq x \leq 1$
d) $x < -2$, $-1 < x < 1$ or $x > 2$
e) $x \leq -1$ or $0 \leq x \leq 3$
f) $-1 < x < -\frac{1}{2}$ or $x > 2$

8. $(-1, 1)$ and $(2, \infty)$

9. **a)** $x^3 + 11x^2 + 18x = 0$
b) Any values of x for which the graph of the corresponding function is above the x-axis ($y = 0$) are solutions to the original inequality.
c) $-9 < x < -2$ or $x > 0$

10. $f(x) = -3(x + 2)(x - 1)(x - 3)^2$

11. **a)**



b) $0 < v < 154.77$ °C
c) 133.78 °C to 139.56 °C

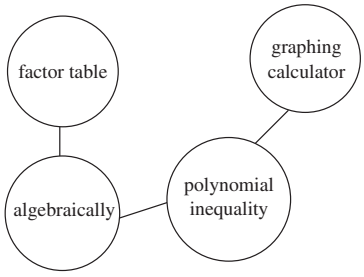
12. **a)** 14 m **c)** $0.3 < t < 2.1$
b) 3.3 s **d)** 1.8 s

13. $V(x) = x(50 - 2x)(30 - 2x)$; $5 < x < 7.19$

14. **a)** Since all the powers are even and the coefficients are positive, the polynomial on the left is always positive.
b) Since all the powers are even and all the coefficients are negative (once all terms are brought to the left), the polynomial on the left is always negative.

15. You cannot divide by a variable expression because you do not know whether it is positive, negative, or zero.
The correct solution is $x < -1$ or $x > 4$.

16. Answers may vary. For example:



17. **a)** $-4 < x < -3$ or $-2 < x < 3$
b) $-1 < x < 0$ or $x > 5$

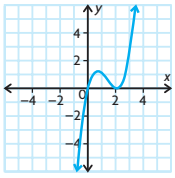
18. $x < -1$ or $x > 2$

Lesson 4.4, pp. 235–237

1. **a)** positive on (0, 1), (4, 7), (10, 15.5), (19, 20); negative on (1, 4), (7, 10), (15.5, 19); zero at $x = 1$, 4, 7, 10, 15.5, and 19
b) A positive slope means the cyclist's elevation is increasing, a negative slope means it is decreasing, and a zero slope means the cyclist's elevation is transitioning from increasing to decreasing or vice versa.

2. **a)** **i)** 6 **ii)** 12 **iii)** 18
b) about 12
c) The graph is increasing on (2, 6).
d) -6
e) about -6

3. **a)** about 0
b) It indicates that $x = 2$ is a turning point in the graph.
c)

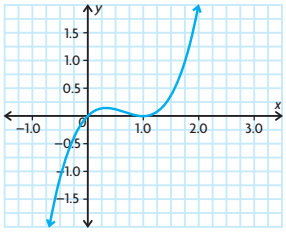


4. **a)** 3
b) Answers may vary. For example, $x = 4.5$, 3.

5. **a)** 3 **c)** $-\frac{1}{10}$ **e)** $\frac{28}{3}$
b) 17 **d)** -7 **f)** 0

6. **a)** 3 **c)** about $-\frac{1}{9}$ **e)** about 5.5
b) about 14 **d)** about -6 **f)** 0

7.



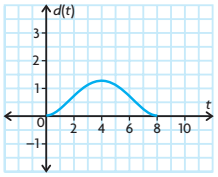
Rate of change is positive on $\left(-\infty, \frac{1}{3}\right)$ and $(1, \infty)$, negative on $\left(\frac{1}{3}, 1\right)$, and zero at $x = \frac{1}{3}$ and 1.

8. **a)** -55 m/s
b) about -20 m/s

9. **a)** about 2
b) -2
c) $y = 2x - 4$

10. **a)** about 10 m/s
b) about -50 m/s
c) 0 m/s

11. **a)**



The rate is positive for $t \in (0, 4)$, negative for $t \in (4, 8)$, and zero at $t = 0$, 4 and 8.
b) When the rate of change is zero, the boat stops.
c) When the rate of change is negative, the boat is headed back to the dock.

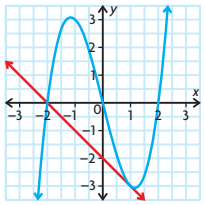
12. At $(-3, 0)$, instantaneous rate $\doteq -96$; at $(1, 0)$, instantaneous rate $\doteq 0$; at $(3, 0)$, instantaneous rate $\doteq 24$; at $(-1, 0)$, instantaneous rate $\doteq 24$

13. **a)** about 5 **c)** $2x + 3$
b) $2x + 3 + h$ **d)** $2(1) + 3 = 5$

14. When the instantaneous rate of change is zero, the function potentially has a local maximum or a local minimum. If the rate is positive to the left and negative to the right, it has a local maximum. If the rate is negative to the left and positive to the right, it has a local minimum.

15. **a)** Rate of change and $f(5)$ are both approximately 148.4.
b) Answers may vary. For example, the instantaneous rate of change at $x = 1$ is 2.7; at $x = 3$, it is 20.1; and at $x = 4$, it is 54.6.
c) The instantaneous rate of change of e^x for any value of x is e^x.

16. **a)** about -1
b) $y = -x - 2$
c) $(-2, 0)$



17. $x = -0.53$, 2.53

Chapter Review, pp. 240–241

1. **a)** ± 3 **c)** 0, -2, 1
b) $\frac{1}{2}$, -2 **d)** ± 1, 2

2. 0, 2, $\frac{2}{3}$, $\frac{4}{5}$

3. **a)** $f(x) = (x - 1)(x - 2)(x + 1)(x + 2)$ or $f(x) = x^4 - 5x^2 + 4$
b) 48, 3.10

4. 2 cm by 2 cm or 7.4 cm by 7.4 cm

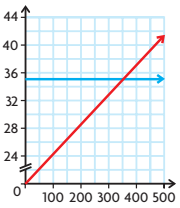
5. **a)** The given information states that the model is valid between 1985 and 1995, so it can be used for 1993, but not 2005.
b) Set $C(t) = 1500$ (since the units are in thousands) and solve using a graphing calculator.
c) Sales reach 1.5 million in the 8th year after 1985, so in 1993.

6. **a)** Answers may vary. For example, $2x + 1 > 17$
b) Answers may vary. For example, $3x - 4 \geq -16$
c) Answers may vary. For example, $2x + 3 \leq -21$
d) Answers may vary. For example, $-19 < 2x - 1 < -3$

7. **a)** $x \in \left(\frac{25}{2}, \infty\right)$
b) $x \in \left[-\frac{23}{8}, \infty\right)$
c) $x \in (-\infty, 2)$
d) $x \in (-\infty, 3]$

8. **a)** $\{x \in \mathbf{R} \mid -2 < x < 4\}$
b) $\{x \in \mathbf{R} \mid -1 \leq x \leq 0\}$
c) $\{x \in \mathbf{R} \mid -3 \leq x \leq 5\}$
d) $\{x \in \mathbf{R} \mid -6 < x < -2\}$

9. **a)** The second plan is better if one calls more than 350 min per month.
b)



10. **a)** $-1 < x < 2$
b) $x \leq -\frac{3}{2}$ or $x \geq 5$
c) $x < -\frac{5}{2}$ or $1 < x < 7$
d) $x \leq -4$ or $1 \leq x \leq 5$

11. negative when $x \in (0, 5)$, positive when $x \in (-\infty, -2)$, $(-2, 0)$, $(5, \infty)$

12. $x \leq -3.81$

13. between January 1993 and March 1994 and between October 1995 and October 1996

14. **a)** average = 7, instantaneous $\doteq$ 8
b) average = 13, instantaneous $\doteq$ 15
c) average = 129, instantaneous $\doteq$ 145
d) average = -464, instantaneous $\doteq -485$

15. positive when $-1 < x < 1$, negative when $x < -1$ or $x > 1$, and zero at $x = -1, 1$

16. **a)** $t \doteq 2.2$ s
b) -11 m/s
c) about -22 m/s

17. **a)** about 57.002
b) about 56.998
c) Both approximate the instantaneous rate of change at $x = 3$.

18. **a)** male: $f(x) = 0.001x^3 - 0.162x^2 + 3.394x + 72.365$; female: $g(x) = 0.0002x^3 - 0.026x^2 + 1.801x + 14.369$
b) More females than males will have lung cancer in 2006.
c) The rate was changing faster for females, on average. Looking only at 1975 and 2000, the incidence among males increased only 5.5 per 100 000, while the incidence among females increased by 31.7.
d) Between 1995 and 2000, the incidence among males decreased by 6.1 while the incidence among females increased by 5.6. Since 1998 is about halfway between 1995 and 2000, an estimate for the instantaneous rate of change in 1998 is the average rate of change from 1995 to 2000. The two rates of change are about the same in magnitude, but the rate for females is positive, while the rate for males is negative.

Chapter Self-Test, p. 242

1. $1, \frac{3}{2}, -2$

2. **a)** positive when $x < -2$ and $0 < x < 2$, negative when $-2 < x < 0$ and $x > 2$, and zero at $-2, 0, 2$
b) positive when $-1 < x < 1$, negative when $x < -1$ or $1 < x$, and zero at $x = -1, 1$
c) -1

3. **a)** Cost with card: $50 + 5n$; Cost without card: $12n$
b) at least 8 pizzas

4. **a)** $x < \frac{1}{2}$
b) $-2 \leq x \leq 1$
c) $-2 < x < -1$ or $x > 5$
d) $x \geq -3$

5. **a)** 15 m
b) 4.6 s
c) -3 m/s

6. **a)** about 5 **b)** (1, 3) **c)** $y = 5x - 2$

7. Since all the exponents are even and all the coefficients are positive, all values of the function are positive and greater than or equal to 4 for all real numbers x.

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9. 2 cm by 2 cm by 15 cm

Chapter 5

Getting Started, pp. 246–247

1. **a)** $(x - 5)(x + 2)$
b) $3(x + 5)(x - 1)$
c) $(4x - 7)(4x + 7)$
d) $(3x - 2)(3x - 2)$
e) $(a - 3)(3a + 10)$
f) $(2x + 3y)(3x - 7y)$

2. **a)** $3 - 2s$
b) $\frac{n^3}{3m}, m, n \neq 0$
c) $3x^2 - 4x - 1, x \neq 0$
d) $\frac{1}{5x - 2}, x \neq \frac{2}{5}$
e) $-\frac{x + 6}{3 + x}, x \neq -3, 3$
f) $\frac{a - b}{a - 3b}, a \neq -5b, \frac{3b}{2}$

3. **a)** $\frac{7}{15}$
b) $\frac{6}{x}, x \neq 0$
c) $\frac{-4x^2 + 20x - 6}{x - 3}, x \neq -2, 3$
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4. **a)** $1\frac{11}{21}$
b) $\frac{19x}{12}$
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e) $\frac{2x + 10 + y}{x^2 - 25}, x \neq 5, -5$
f) $\frac{-2a + 50}{(a + 3)(a - 5)(a + 3)}, x \neq -3, 4, 5$

5. **a)** $x = 6$
b) $x = 2$
c) $x = 3$
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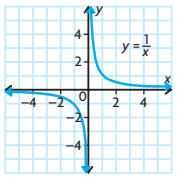
6.



vertical: $x = 0$; horizontal: $y = 0$;
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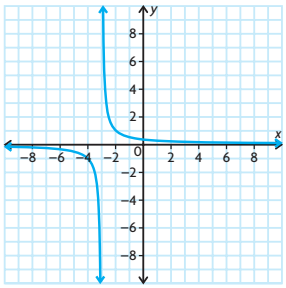
$R = \{y \in \mathbf{R} \mid y \neq 0\}$

7. **a)** translated three units to the left



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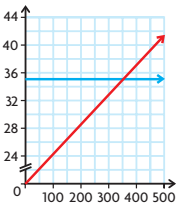

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Chapter Self-Test, p. 242

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Chapter 5

Getting Started, pp. 246–247

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f) $\frac{-2a + 50}{(a + 3)(a - 5)(a + 3)}, x \neq -3, 4, 5$

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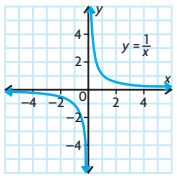

vertical: $x = 0$; horizontal: $y = 0$;
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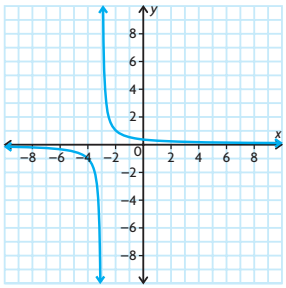
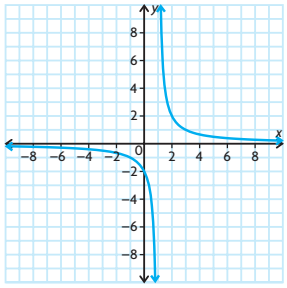
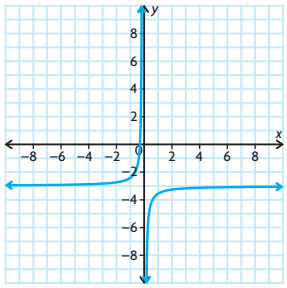
7. **a)** translated three units to the left



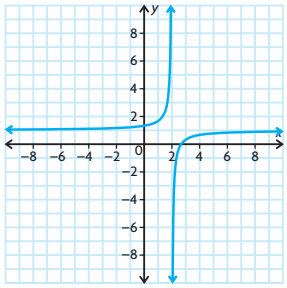
b) vertical stretch by a factor of 2 and a horizontal translation 1 unit to the right



c) reflection in the x-axis, vertical compression by a factor of $\frac{1}{2}$, and a vertical translation 3 units down



d) reflection in the x-axis, vertical compression by a factor of $\frac{2}{3}$, horizontal translation 2 units right, and a vertical translation 1 unit up



8. Factor the expressions in the numerator and the denominator. Simplify each expression as necessary. Multiply the first expression by the reciprocal of the second.

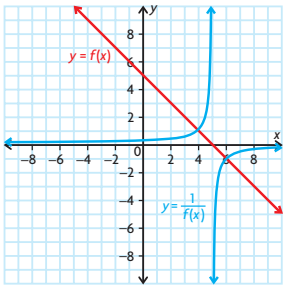
$$\frac{-3(3y-2)}{2(3y+2)}$$

Lesson 5.1, pp. 254–257

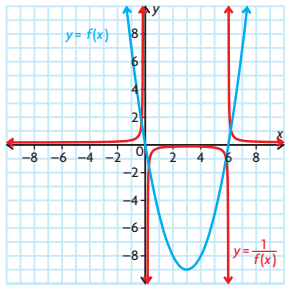
1. **a)** C; The reciprocal function is F.
b) A; The reciprocal function is E.
c) D; The reciprocal function is B.
d) F; The reciprocal function is C.
e) B; The reciprocal function is D.
f) E; The reciprocal function is A.

2. **a)** $x = 6$
b) $x = -\frac{4}{3}$
c) $x = 5$ and $x = -3$
d) $x = -\frac{5}{2}$ and $x = \frac{5}{2}$
e) no asymptotes
f) $x = -1.5$ and $x = -1$

3. **a)**



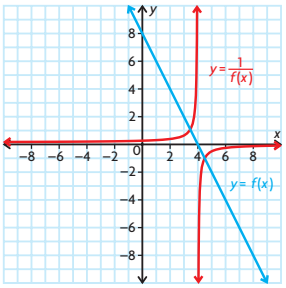
b)



4. **a)**

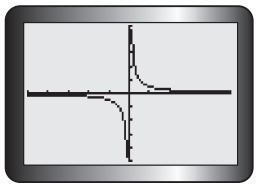
x	$f(x)$	$\frac{1}{f(x)}$
-4	16	$\frac{1}{16}$
-3	14	$\frac{1}{14}$
-2	12	$\frac{1}{12}$
-1	10	$\frac{1}{10}$
0	8	$\frac{1}{8}$
1	6	$\frac{1}{6}$
2	4	$\frac{1}{4}$
3	2	$\frac{1}{2}$
4	0	undefined
5	-2	$-\frac{1}{2}$
6	-4	$-\frac{1}{4}$
7	-6	$-\frac{1}{6}$

b)

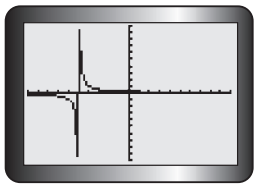


c) $f(x) = -2x + 8, y = \frac{1}{-2x+8}$

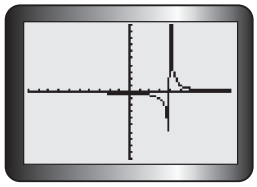
5. **a)** $y = \frac{1}{2x}$; vertical asymptote at $x = 0$



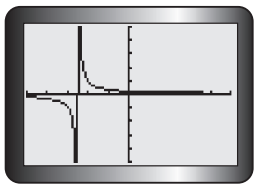
b) $y = \frac{1}{x+5}$; vertical asymptote at $x = -5$



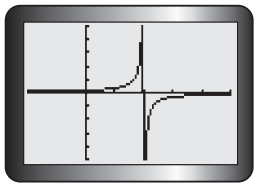
c) $y = \frac{1}{x-4}$; vertical asymptote at $x = 4$



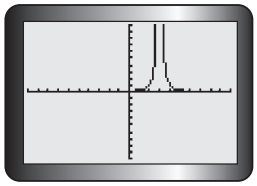
d) $y = \frac{1}{2x+5}$; vertical asymptote at $x = -\frac{5}{2}$



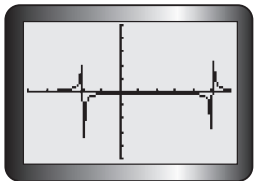
e) $y = \frac{1}{-3x+6}$; vertical asymptote at $x = 2$



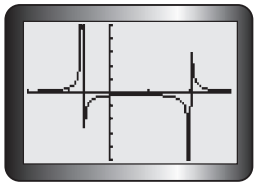
f) $y = \frac{1}{(x-3)^2}$; vertical asymptote at $x = 3$



g) $y = \frac{1}{x^2 - 3x - 10}$; vertical asymptotes at $x = -2$ and $x = 5$



h) $y = \frac{1}{3x^2 - 4x - 4}$; vertical asymptotes at $x = -\frac{2}{3}$ and $x = 2$



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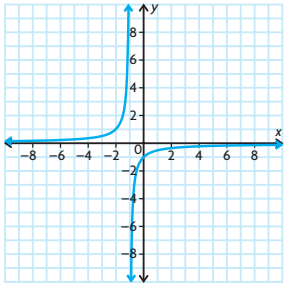
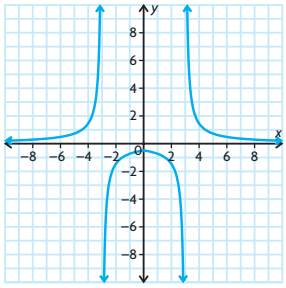


b)



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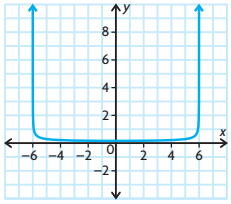

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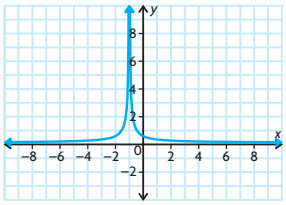

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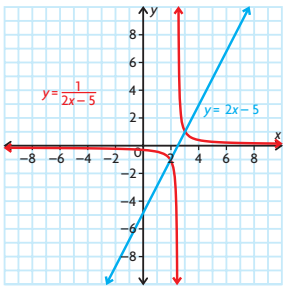

$D = \left\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\right\}$,
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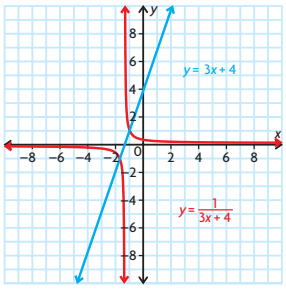
b)



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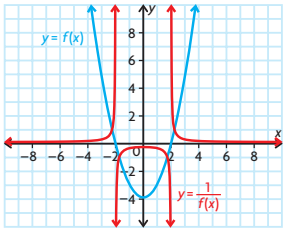
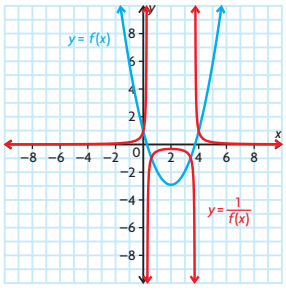


b)



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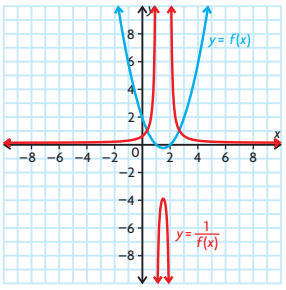

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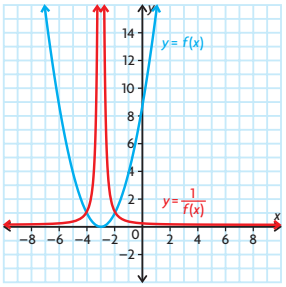

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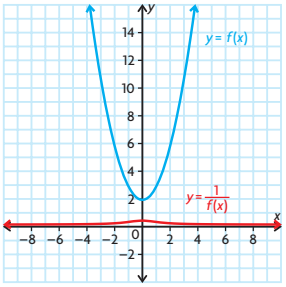

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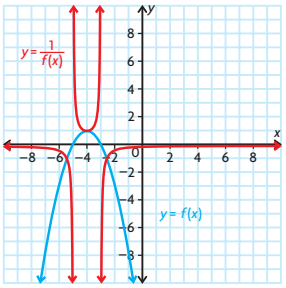
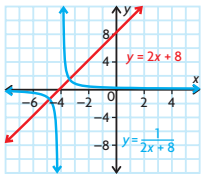
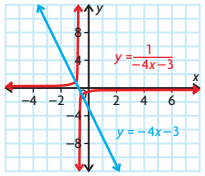


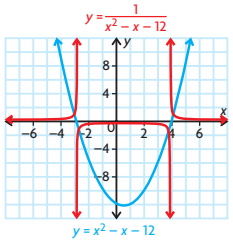
9. **a)** $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R}\}$
y-intercept $= 8$
x-intercept $= -4$
negative on $(-\infty, -4)$
positive on $(-4, -\infty)$
increasing on $(-\infty, \infty)$
equation of reciprocal $= \frac{1}{2x + 8}$



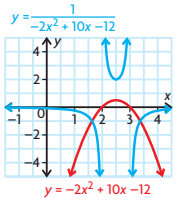
b) $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R}\}$
y-intercept $= -3$
x-intercept $= -\frac{3}{4}$
positive on $\left(-\infty, -\frac{3}{4}\right)$
negative on $\left(-\frac{3}{4}, \infty\right)$
decreasing on $(-\infty, \infty)$
equation of reciprocal $= \frac{1}{-4x - 3}$



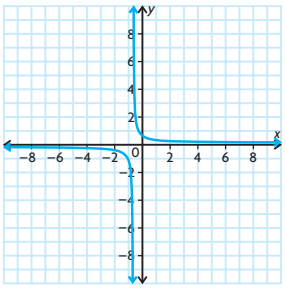
c) $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} | y \geq -12.25\}$
y-intercept $= 12$
x-intercepts $= 4, -3$
decreasing on $(-\infty, 0.5)$
increasing on $(0.5, \infty)$
positive on $(-\infty, -3)$ and $(4, \infty)$
negative on $(-3, 4)$
equation of reciprocal $= \frac{1}{x^2 - x - 12}$



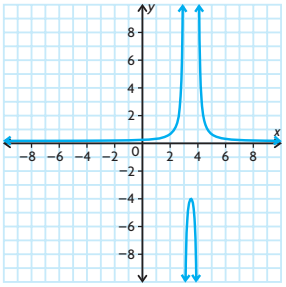
d) $D = \{x \in \mathbf{R}\}$
$R = \{y \in \mathbf{R} | y \leq 2.5\}$
y-intercept $= -12$
x-intercepts $= 3, 2$
increasing on $(-\infty, 2.5)$
decreasing on $(2.5, \infty)$
negative on $(-\infty, 2)$ and $(3, \infty)$
positive on $(2, 3)$
equation of reciprocal $= \frac{1}{-2x^2 + 10x - 12}$



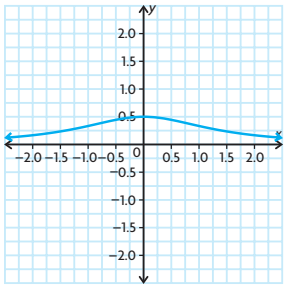
10. Answers may vary. For example, a reciprocal function creates a vertical asymptote when the denominator is equal to 0 for a specific value of x. Consider $\frac{1}{ax + b}$. For this expression, there is always some value of x that is $\frac{-b}{a}$ that will result in a vertical asymptote for the function. This is a graph of $y = \frac{1}{3x + 2}$ and the vertical asymptote is at $x = -\frac{2}{3}$.



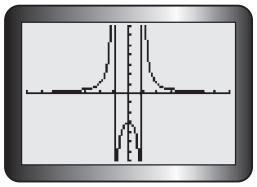
Consider the function $\frac{1}{(x - 3)(x - 4)}$. The graph of the quadratic function in the denominator crosses the x-axis at 3 and 4 and therefore will have vertical asymptotes at 3 and 4 in the graph of the reciprocal function.



However, a quadratic function, such as $x^2 + c$, which has no real zeros, will not have a vertical asymptote in the graph of its reciprocal function. For example, this is the graph of $y = \frac{1}{x^2 + 2}$.

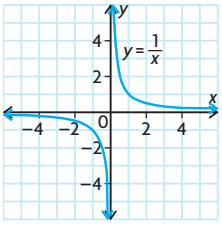


11. $y = \frac{3}{x^2 - 1}$



12. **a)** 500
b) $t = 2$
c) $t = 10\,000$
d) If you were to use a value of t that was less than one, the equation would tell you that the number of bacteria was increasing as opposed to decreasing. Also, after time $t = 10\,000$, the formula indicates that there is a smaller and smaller fraction of 1 bacteria left.
e) $D = \{x \in \mathbf{R} | 1 < x < 10\,000\}$,
$R = \{y \in \mathbf{R} | 1 < y < 10\,000\}$

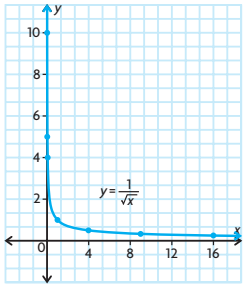
13. **a)**



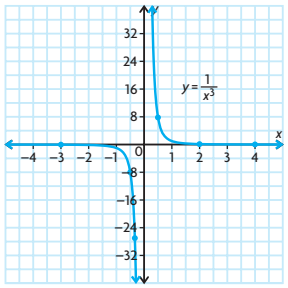
$D = \{x \in \mathbf{R} | x \neq -n\}$,
$R = \{y \in \mathbf{R} | y \neq 0\}$
b) The vertical asymptote occurs at $x = -n$. Changes in n in the $f(x)$ family cause changes in the y-intercept—an increase in n causes the intercept to move up the y-axis and a decrease causes it to move down the y-axis. Changes in n in the $g(x)$ family cause changes in the vertical asymptote of the function—an increase in n causes the asymptote to move down the x-axis and a decrease in n causes it to move up the x-axis.
c) $x = 1 - n$ and $x = -1 - n$

14. Answers may vary. For example:
1) Determine the zero(s) of the function $f(x)$—these will be the asymptote(s) for the reciprocal function $g(x)$.
2) Determine where the function $f(x)$ is positive and where it is negative—the reciprocal function $g(x)$ will have the same characteristics.
3) Determine where the function $f(x)$ is increasing and where it is decreasing—the reciprocal function $g(x)$ will have opposite characteristics.

15. **a)**



b)



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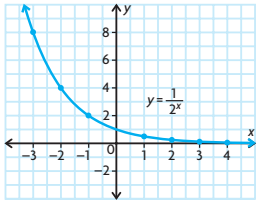
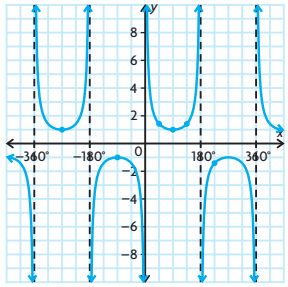


d)



16. $y = \dfrac{1}{x + 4} - 1$

Lesson 5.2, p. 262

1. **a)** A; The function has a zero at 3 and the reciprocal function has a vertical asymptote at $x = 3$. The function is positive for $x < 3$ and negative for $x > 3$.
b) C; The function in the numerator factors to $(x + 3)(x - 3)$. $(x - 3)$ factors out of both the numerator and the denominator. The equation simplifies to $y = x + 3$, but has a hole at $x = 3$.
c) F; The function in the denominator has a zero at $x = -3$, so there is a vertical asymptote at $x = -3$. The function is always positive.
d) D; The function in the denominator has zeros at $y = 1$ and $y = -3$. The rational function has vertical asymptotes at $x = 1$ and $x = -3$.
e) B; The function has no zeros and no vertical asymptotes or holes.
f) E; The function in the denominator has a zero at $x = 3$ and the rational function has a vertical asymptote at $x = 3$. The degree of the numerator is exactly 1 more than the degree of the denominator, so the graph has an oblique asymptote.

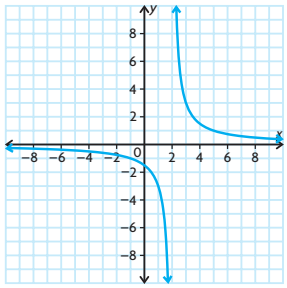
2. **a)** vertical asymptote at $x = -4$; horizontal asymptote at $y = 1$
b) vertical asymptote at $x = -\frac{3}{2}$; horizontal asymptote at $y = 0$
c) vertical asymptote at $x = 6$; horizontal asymptote at $y = 2$
d) hole at $x = -3$
e) vertical asymptotes at $x = -3$ and 5; horizontal asymptote at $y = 0$
f) vertical asymptote at $x = -1$; horizontal asymptote at $y = -1$
g) hole at $x = 2$
h) vertical asymptote at $x = \frac{5}{2}$; horizontal asymptote at $y = -2$
i) vertical asymptote at $x = -\frac{1}{4}$; horizontal asymptote at $y = 1$
j) vertical asymptote at $x = 4$; hole at $x = -4$; horizontal asymptote at $y = 0$
k) vertical asymptote at $x = \frac{3}{5}$; horizontal asymptote at $y = \frac{1}{5}$
l) vertical asymptote at $x = 4$; horizontal asymptote at $y = -\frac{3}{2}$

3. Answers may vary. For example:
a) $y = \dfrac{x - 1}{x^2 + x - 2}$
b) $y = \dfrac{1}{x^2 - 4}$
c) $y = \dfrac{x^2 - 4}{x^2 + 3x + 2}$
d) $y = \dfrac{2x}{x + 1}$
e) $y = \dfrac{x^3}{x^2 + 5}$

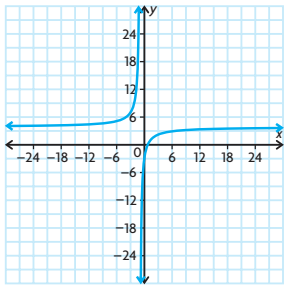
Lesson 5.3, pp. 272–274

1. **a)** A **c)** D
b) C **d)** B

2. **a)** $x = 2$
b) As $x \to 2$ from the right, the values of $f(x)$ get larger. As $x \to 2$ from the left, the values become larger in magnitude but are negative.
c) $y = 0$
d) As $x \to -\infty$ and as $x \to \infty$, $f(x) \to 0$.
e) D $= \{x \in \mathbf{R} \mid x \neq 3\}$
R $= \{y \in \mathbf{R} \mid y \neq 0\}$
f) positive: $(2, \infty)$
negative: $(-\infty, 2)$
g)



3. **a)** $x = -1$
b) As $x \to -1$ from the left, $y \to \infty$. As $x \to -1$ from the right, $y \to -\infty$.
c) $y = 4$
d) As $x \to \pm\infty$, $f(x)$ gets closer and closer to 4.
e) D $= \{x \in \mathbf{R} \mid x \neq -1\}$
R $= \{y \in \mathbf{R} \mid y \neq 4\}$
f) positive: $(-\infty, -1)$ and $\left(\frac{3}{4}, -\infty\right)$
negative: $\left(-1, \frac{3}{4}\right)$
g)



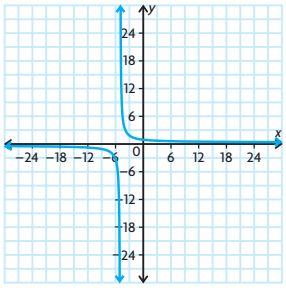
4. **a)** $x = -3$; As $x = -3$, $y = -\infty$ on the left.
As $x = -3$, $y = \infty$ on the right.

b) $x = 5$; As $x = 5$, $y = -\infty$ on the left.
As $x = 5$, $y = \infty$ on the right.

c) $x = \frac{1}{2}$; As $x = \frac{1}{2}$, $y = -\infty$ on the left.
As $x = \frac{1}{2}$, $y = \infty$ on the right.

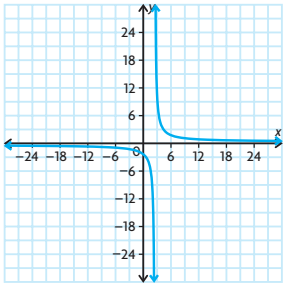
d) $x = -\frac{1}{4}$; As $x = -\frac{1}{4}$, $y = -\infty$ on the left.
As $x = -\frac{1}{4}$, $y = \infty$ on the right.

5. **a)** vertical asymptote at $x = -5$
horizontal asymptote at $y = 0$
$D = \{x \in \mathbf{R} \mid x \neq -5\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
y-intercept $= \frac{3}{5}$
$f(x)$ is negative on $(-\infty, -5)$ and positive on $(-5, \infty)$.



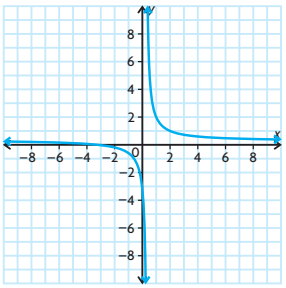
The function is decreasing on $(-\infty, -5)$ and on $(-5, \infty)$. The function is never increasing.

b) vertical asymptote at $x = \frac{5}{2}$
horizontal asymptote at $y = 0$
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{5}{2}\right\}$
$R = \{y \in \mathbf{R} \mid y \neq 0\}$
y-intercept $= -2$
$f(x)$ is negative on $\left(-\infty, \frac{5}{2}\right)$ and positive on $\left(\frac{5}{2}, \infty\right)$.



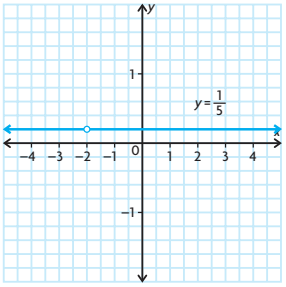
The function is decreasing on $\left(-\infty, \frac{5}{2}\right)$ and on $\left(\frac{5}{2}, \infty\right)$. The function is never increasing.

c) vertical asymptote at $x = \frac{1}{4}$
horizontal asymptote at $y = \frac{1}{4}$
$D = \left\{x \in \mathbf{R} \mid x \neq \frac{1}{4}\right\}$
$R = \left\{y \in \mathbf{R} \mid y \neq \frac{1}{4}\right\}$
x-intercept $= -5$
y-intercept $= -1$
$f(x)$ is positive on $(-\infty, -5)$ and $\left(\frac{1}{4}, \infty\right)$ and negative on $\left(-5, \frac{1}{4}\right)$.



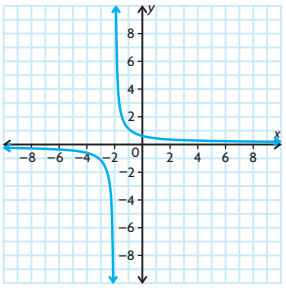
The function is decreasing on $\left(-\infty, \frac{1}{4}\right)$ and on $\left(\frac{1}{4}, \infty\right)$. The function is never increasing.

d) hole $x = -2$
$D = \{x \in \mathbf{R} \mid x \neq -2\}$
$R = \left\{y = \frac{1}{5}\right\}$
y-intercept $= \frac{1}{5}$
The function will always be positive.

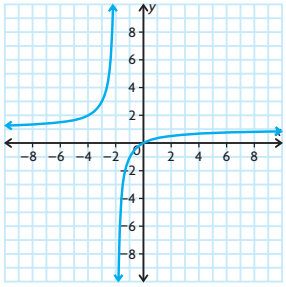


The function is neither increasing nor decreasing; it is constant.

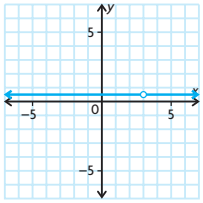
6. **a)** Answers may vary. For example:
$f(x) = \frac{1}{x + 2}$



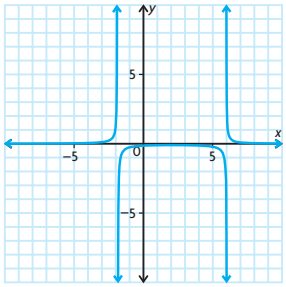
b) Answers may vary. For example:
$y = \frac{x}{x + 2}$



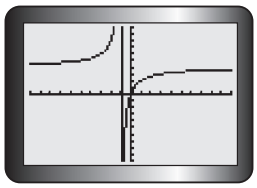
c) Answers may vary. For example:
$f(x) = \frac{x - 3}{2x - 6}$

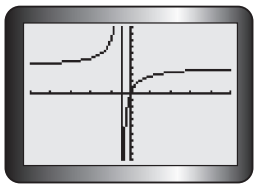


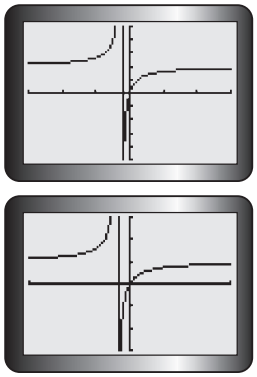
d) Answers may vary. For example:
$f(x) = \frac{1}{x^2 - 4x - 12}$



7. **a)**







The equation has a general vertical asymptote at $x = -\frac{1}{n}$. The function has a general horizontal asymptote at $y = \frac{8}{n}$. The vertical asymptotes are $-\frac{1}{8}$, $-\frac{1}{4}$, $-\frac{1}{2}$, and -1. The horizontal asymptotes are 8, 4, 2, and 1. The function contracts as n increases. The function is always increasing. The function is positive on $\left(-\infty, -\frac{17}{n}\right)$ and $\left(\frac{3}{10}, \infty\right)$. The function is negative on $\left(-\frac{17}{n}, \frac{3}{10}\right)$.

b) The horizontal and vertical asymptotes both approach 0 as the value of n increases; the x- and y-intercepts do not change, nor do the positive and negative characteristics or the increasing and decreasing characteristics.

c) The vertical asymptote becomes $x = \frac{17}{n}$ and the horizontal becomes $x = -\frac{10}{n}$. The function is always increasing. The function is positive on $\left(-\infty, \frac{3}{10}\right)$ and $\left(\frac{17}{n}, \infty\right)$. The function is negative on $\left(\frac{3}{10}, \frac{17}{n}\right)$. The rest of the characteristics do not change.

8. $f(x)$ will have a vertical asymptote at $x = 1$; $g(x)$ will have a vertical asymptote at $x = -\frac{3}{2}$. $f(x)$ will have a horizontal asymptote at $x = 3$; $g(x)$ will have a vertical asymptote at $x = \frac{1}{2}$.

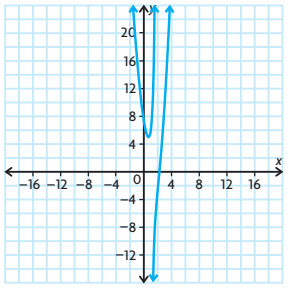
9. **a)** \$27 500
b) \$40 000
c) \$65 000
d) No, the value of the investment at $t = 0$ should be the original value invested.
e) The function is probably not accurate at very small values of t because as $t \to 0$ from the right, $x \to \infty$.
f) \$15 000

10. The concentration increases over the 24 h period and approaches approximately 1.89 mg/L.

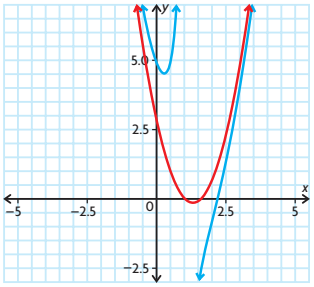
11. Answers may vary. For example, the rational functions will all have vertical asymptotes at $x = -\frac{d}{c}$. They will all have horizontal asymptotes at $y = \frac{a}{c}$. They will intersect the y-axis at $y = \frac{b}{d}$. The rational functions will have an x-intercept at $x = -\frac{b}{a}$.

12. Answers may vary. For example, $f(x) = \frac{2x^2}{2 + x}$.

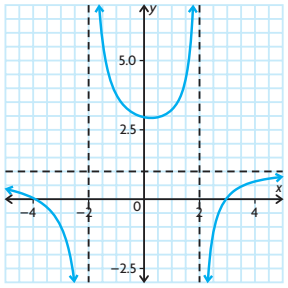
13. $f(x) = 2x^2 - 5x + 3 - \frac{2}{x - 1}$
As $x \to \pm\infty, f(x) \to \infty$.



vertical asymptote: $x = 1$; oblique asymptote: $y = 2x^2 - 5x + 3$



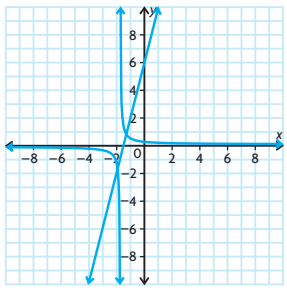
14. **a)** $f(x)$
b) $g(x)$ and $h(x)$
c) $g(x)$
d)



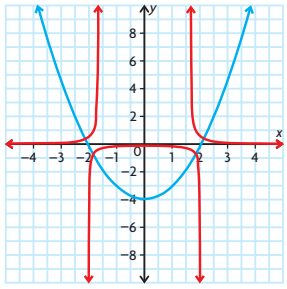
Mid-Chapter Review, p. 277

1. **a)** $\frac{1}{x - 3}$; $x = 3$
b) $\frac{1}{-4q + 6}$; $q = \frac{3}{2}$
c) $\frac{1}{z^2 + 4z - 5}$; $z = -5$ and 1
d) $\frac{1}{6d^2 + 7d - 3}$; $d = \frac{1}{3}$ and $-\frac{3}{2}$

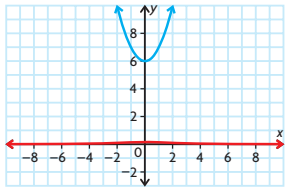
2. **a)** D $= \{x \in \mathbf{R}\}$; R $= \{x \in \mathbf{R}\}$; y-intercept $= 6$; x-intercept $= -\frac{3}{2}$; negative on $\left(-\infty, -\frac{3}{2}\right)$; positive on $\left(-\frac{3}{2}, \infty\right)$; increasing on $(-\infty, \infty)$



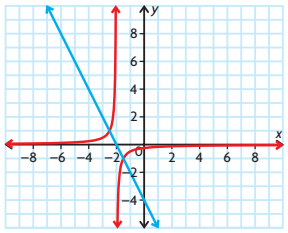
b) D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R} \mid y > -4\}$; y-intercept $= -4$; x-intercepts are 2 and -2; decreasing on $(-\infty, 0)$; increasing $(0, \infty)$; positive on $(-\infty, -2)$ and $(2, \infty)$; negative on $(-2, 2)$



c) D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R} \mid y > 6\}$; no x-intercepts; function will never be negative; decreasing on $(-\infty, 0)$; increasing on $(0, \infty)$



d) D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R}\}$; x-intercept $= -2$; function is always decreasing; positive on $(-\infty, -2)$; negative on $(-2, \infty)$

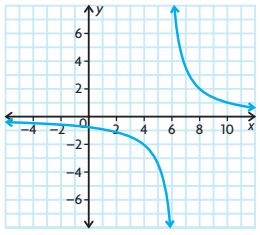


3. Answers may vary. For example: (1) Hole: Both the numerator and the denominator contain a common factor, resulting in $\frac{0}{0}$ for a specific value of x. (2) Vertical asymptote: A value of x causes the denominator of a rational function to be 0. (3) Horizontal asymptote: A horizontal asymptote is created by the ratio between the numerator and the denominator of a rational function as the function $\rightarrow \infty$ and $-\infty$. A continuous rational function is created when the denominator of the rational function has no zeros.

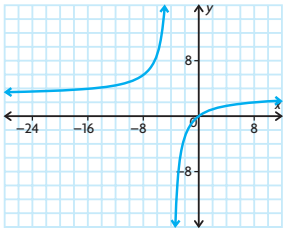
4. **a)** $x = 2$; vertical asymptote
b) hole at $x = 1$
c) $x = -\frac{1}{2}$; horizontal asymptote
d) $x = 6$; oblique asymptote
e) $x = -5$ and $x = 3$; vertical asymptotes

5. $y = \dfrac{x}{x - 2}, y = 1; y = \dfrac{-7x}{4x + 2}, y = \dfrac{-7}{4}; y = \dfrac{1}{x^2 + 2x - 15}, x = 0$

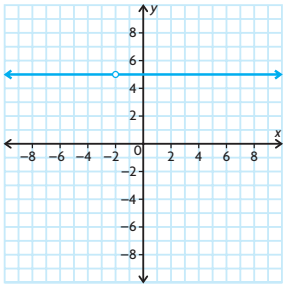
6. **a)** vertical asymptote: $x = 6$; horizontal asymptote: $y = 0$; no x-intercept; y-intercept: $-\frac{5}{6}$; negative when the denominator is negative; positive when the numerator is positive; $x - 6$ is negative on $x < 6$; $f(x)$ is negative on $(-\infty, 6)$ and positive on $(6, \infty)$; function is always decreasing



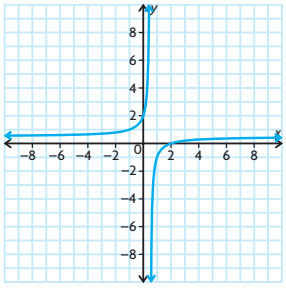
b) vertical asymptote: $x = -4$; horizontal asymptote: $y = 3$; x-intercept: $x = 0$; y-intercept: $f(0) = 0$; function is always increasing; positive on $(-\infty, -4)$ and $(0, \infty)$; negative on $(-4, 0)$



c) straight, horizontal line with a hole at $x = -2$; always positive and never increases or decreases



d) vertical asymptote: $x = \frac{1}{2}$; horizontal asymptote: $y = \frac{1}{2}$; x-intercept: $x = 2$; y-intercept: $f(0) = 5$; function is always increasing



7. Answers may vary. For example: Changing the function to $y = \frac{7x + 6}{x + 1}$ changes the graph. The function now has a vertical asymptote at $x = -1$ and still has a horizontal asymptote at $y = 7$. However, the function is now constantly increasing instead of decreasing. The new function still has an x-intercept at $x = -\frac{6}{7}$, but now has a y-intercept at $y = 6$.

8. $n = \frac{1}{3}; m = 35$

9. Answers may vary. For example, $f(x) = \frac{4x + 8}{x + 2}$.
The graph of the function will be a horizontal line at $y = 4$ with a hole at $x = -2$.

Lesson 5.4, pp. 285–287

1. 3; -2; Answers may vary. For example, substituting each value for x in the equation produces the same value on each side of the equation, so both are solutions.

2. **a)** $x = -3$ **c)** $x = -1$ and 2
b) $x = 5$ **d)** $x = -4$

3. **a)** $f(x) = \dfrac{x - 3}{x + 3} - 2$
b) $f(x) = \dfrac{3x - 1}{x} - \dfrac{5}{2}$
c) $f(x) = \dfrac{x - 1}{x} - \dfrac{x + 1}{x + 3}$
d) $f(x) = \dfrac{x - 2}{x + 3} - \dfrac{x - 4}{x + 5}$

4. **a)** $x = -9$ **c)** $x = 3$
b) $x = 2$ **d)** $x = -\frac{1}{2}$

5. **a)** $x = 3$ **d)** $x = 0$
b) $x = \frac{3}{4}$ **e)** $x = \frac{1}{4}$
c) $x = -9$ **f)** $x = -23$

6. **a)** The function will have no real solutions.
b) $x = 3$ and $x = -0.5$
c) $x = -5$
d) $x = 0$ and $x = -1$
e) The original equation has no real solutions.
f) $x = 5$ and $x = 2$

7. **a)** $x = 6$ **d)** $x = 3.25, 20.75$
b) $x = 1.30, 7.70$ **e)** $x = -1.71, 2.71$
c) $x = 10$ **f)** $x = -0.62, 1.62$

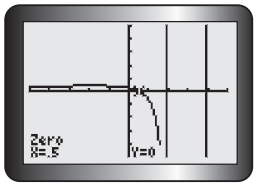
8. **a)** $\dfrac{x + 1}{x - 2} = \dfrac{x + 3}{x - 4}$
Multiply both sides of the equation by the LCD, $(x - 2)(x - 4)$.

$$(x - 2)(x - 4)\left(\frac{x + 1}{x - 2}\right) = (x - 2)(x - 4)\left(\frac{x + 3}{x - 4}\right)$$

$(x - 4)(x + 1) = (x - 2)(x + 3)$
Simplify. $x^2 - 3x - 4 = x^2 + x - 6$
Simplify the equation so that 0 is on one side of the equation.
$x^2 - x^2 - 3x - x - 4 + 6 = x^2 - x^2 + x - x - 6 + 6$
$-4x + 2 = 0$
$-2(2x - 1) = 0$
Since the product is equal to 0, one of the factors must be equal to 0. It must be $2x - 1$ because 2 is a constant.
$2x - 1 = 0$
$2x - 1 + 1 = 0 + 1$
$2x = 1$
$\dfrac{2x}{2} = \dfrac{1}{2}$
$x = \dfrac{1}{2}$

b) $\frac{\frac{1}{2}+1}{\frac{1}{2}-2} = -1$ and $\frac{\frac{1}{2}+3}{\frac{1}{2}-4} = -1$

c)



9. $w = 9.271$

10. Machine A = 25.8 min; Machine B = 35.8 min

11. 75; $4.00

12. **a)** After 6666.67 s

b) The function appears to approach 9 kg/m^3 as time increases.

13. **a)** Tom = 4 min; Carl = 5 min; Paco = 2 min

b) 6.4 min

14. Answers may vary. For example, you can use either algebra or graphing technology to solve a rational equation. With algebra, solving the equation takes more time, but you get an exact answer. With graphing technology, you can solve the equation quickly, but you do not always get an exact answer.

15. $x = -3.80, -1.42, 0.90, 4.33$

16. **a)** $x = 0.438$ and 1.712

b) $(0, 0.438)$ and $(1.712, \infty)$

Lesson 5.5, pp. 295–297

1. **a)** $(\infty, 1)$ and $(3, \infty)$

b) $(-0.5, 1)$ and $(2, \infty)$

2. **a)** Solve the inequality for x.

$$\frac{6x}{x+3} \le 4$$
$$\frac{6x}{x+3} - 4 \le 0$$
$$\frac{6x}{x+3} - 4\frac{x+3}{x+3} \le 0$$
$$\frac{6x - 4x - 12}{x+3} \le 0$$
$$\frac{2x-12}{x+3} \le 0$$
$$\frac{2(x-6)}{x+3} \le 0$$

b)



c) $(-3, 6]$

3. **a)**
$$x + 2 > \frac{15}{x}$$
$$x + 2 - \frac{15}{x} > 0$$
$$\frac{x^2}{x} + \frac{2x}{x} - \frac{15}{x} > 0$$
$$\frac{x^2 + 2x - 15}{x} > 0$$
$$\frac{(x+5)(x-3)}{x} > 0$$

b) negative: $x < -5$ and $0 < x < 3$; positive: $-5 < x < 0, x > 3$

c) $\{x \in \mathbf{R} \mid -5 < x < 0 \text{ or } x > 3\}$ or $(-5, 0)$ or $(3, \infty)$

4. **a)** $5 < x < -4.5$

b) $-7 < x < -5$ and $x > -3$

c) $0 < x < 2$ and $x > 8$

d) $-6.8 \le x < -4$ and $x > 3$

e) $x < -1$ and $-\frac{1}{7} < x < 0$

f) $-1 < x < \frac{7}{8}$ and $x < 4$

5. **a)** $t < -3$ or $1 < t < 4$

b) $-3 \le t \le 2$ or $t > 4$

c) $-\frac{1}{2} < t < \frac{1}{3}$ or $t > \frac{1}{2}$

d) $t < -2$ and $-2 < t < 3$

e) $t < -5$ and $-2 < t < 0$

f) $-1 \le t < 0.25$ and $2 \le t < 9$

6. **a)** $x \in (-\infty, -6)$ or $x \in (-1, 4)$

b) $x \in (3, \infty)$

c) $x \in (-4, -2)$ or $x \in (-1, 2)$

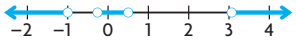
d) $x \in (-\infty, -9)$ or $x \in [-3, -1)$ or $x \in [3, \infty)$

e) $x \in (-2, 0)$ or $x \in (4, \infty)$

f) $x \in (-\infty, -4)$ or $x \in (4, \infty)$

7. **a)** $x < -1, -0.2614 < x < 0.5, x > 3.065$

b)

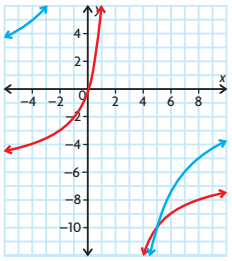


c) Interval notation: $(-\infty, -1)$, $(-0.2614, 0.5)$, $(3.065, \infty)$

Set notation: $\{x \in \mathbf{R} \mid x < -1, -0.2614 < x < 0.5, \text{ or } x > 3.065\}$

8. **a)** $t < 2$ and $t > 5$.

b)



c) It would be difficult to find a situation that could be represented by these rational expressions because very few positive values of x yield a positive value of y.

9. The only values that make the expression greater than 0 are negative. Because the values of t have to be positive, the bacteria count in the tap water will never be greater than that of the pond water.

10. **a)** $\frac{(x^2 - 4x - 5)}{2x < 0}$

b)

	$x < -1$	$-1 < x < 0$	$0 < x < 5$	$x > 5$
$(x - 5)$	−	−	−	+
$(x + 1)$	−	+	+	+
$2x$	−	−	+	+
$\frac{(x-5)(x+1)}{2x}$	−	+	−	+

The inequality is true for $x < -1$ and $0 < x < 5$

11. when $x > 5$

12. **a)** The first inequality can be manipulated algebraically to produce the second inequality.

b) Graph the equation $y = \frac{x+1}{x-1} - \frac{x+3}{x+2}$ and determine when it is negative.

c) The values that make the factors of the second inequality zero are -5, -2, and 1. Determine the sign of each factor in the intervals corresponding to the zeros. Determine when the entire expression is negative by examining the signs of the factors.

13. $[2, 4)$ and $(4, \infty)$

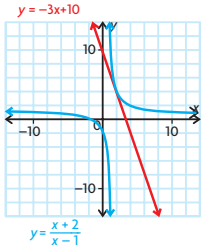
14. $14.48 < x < 165.52$ and $180 < x < 360$

15. $0 < x < 2$

Lesson 5.6, pp. 303–305

1. **a)** -0.5

b)



slope = -3

2. -3

3. -3

4. -1

5. **a)** 0.01

b) -0.3

c) -1.3

d) 6

6. **a)** slope = 286.1; vertical asymptote: $x = -1.5$

b) slope = -2.74; vertical asymptote: $x = -5$

c) slope = 44.65; vertical asymptote: $x = -\frac{5}{3}$

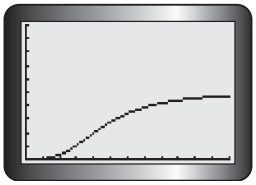
d) slope = -1.26; vertical asymptote: $x = 6$

7. **a)** 0.01
b) 0.34

8. **a)** $R(x) = \dfrac{15x}{2x^2 + 11x + 5}$
b) 0.3, −0.03

9. **a)** $5.67
b) −2

10. **a)** 68.46
b) 94.54
c)



The number of houses that were built increases slowly at first, but rises rapidly between the third and sixth months. During the last six months, the rate at which the houses were built decreases.

11. Answers may vary. For example:
$14 \le x \le 15$; $x = 14.5$

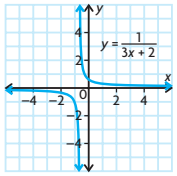
12. **a)** Find $s(0)$ and $s(6)$, and then solve $\dfrac{s(6) - s(0)}{6 - 0}$.
b) The average rate of change over this interval gives the object's speed.
c) To find the instantaneous rate of change at a specific point, you could find the slope of the line that is tangent to the function $s(t)$ at the specific point. You could also find the average rate of change on either side of the point for smaller and smaller intervals until it stabilizes to a constant. It is generally easier to find the instantaneous rate using a graph, but the second method is more accurate.
d) The instantaneous rate of change for a specific time, t, is the acceleration of the object at this time.

13. $y = -0.5x - 2.598$; $y = -0.5x + 2.598$; $y = 4x$

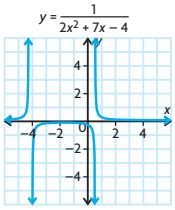
14. The instantaneous rate of change at $(0, 0) = 4$. The rate of change at this rate of change will be 0.

Chapter Review, pp. 308–309

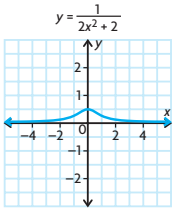
1. **a)** D = $\{x \in \mathbf{R}\}$; R = $\{y \in \mathbf{R}\}$;
x-intercept = $-\frac{2}{3}$; y-intercept = 2;
always increasing;
negative on $\left(-\infty, -\frac{2}{3}\right)$;
positive on $\left(-\frac{2}{3}, \infty\right)$



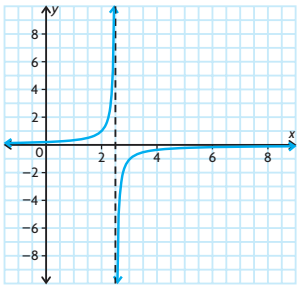
b) D = $\{x \in \mathbf{R}\}$;
R = $\{y \in \mathbf{R} \mid y > -10.125\}$;
x-intercept = 0.5 and −4;
positive on $(-\infty, -4)$ and $(0.5, \infty)$;
negative on $(-4, 0.5)$;
decreasing on $(-\infty, -10.125)$;
increasing on $(-10.125, \infty)$



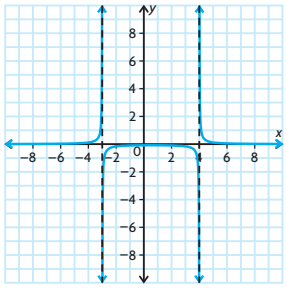
c) D = $\{x \in \mathbf{R}\}$; R = $\{y \in \mathbf{R} \mid y > 2\}$; no x-intercepts; y-intercept = 2;
decreasing on $(-\infty, 0)$;
increasing on $(0, \infty)$; always positive, never negative



2. **a)**



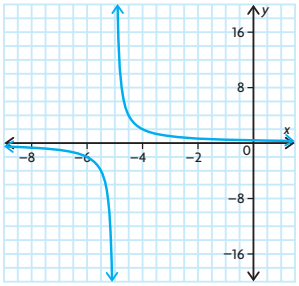
b)



3. **a)** $x = -17$
b) $x = -\frac{3}{5}$; horizontal asymptote; $y = \frac{2}{5}$
c) $x = 0.5$; hole at $x = -11$
d) $x = 1$; oblique asymptote; $y = 3x + 3$

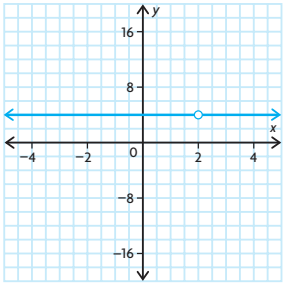
4. The locust population increased during the first 1.75 years, to reach a maximum of 1 248 000. The population gradually decreased until the end of the 50 years, when the population was 128 000.

5. **a)** x-intercept = 2:
horizontal asymptote: $y = 0$;
y-intercept = $\frac{2}{5}$:
vertical asymptote: $x = -5$;



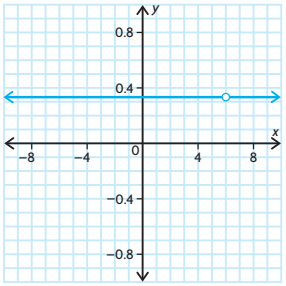
The function is never increasing and is decreasing on $(-\infty, -5)$ and $(-5, \infty)$.
D = $\{x \in \mathbf{R} \mid x \neq -5\}$;
negative for $x < -5$;
positive for $x > -5$

b) D = $\{x \in \mathbf{R} \mid x \neq 2\}$; no x-intercept; y-intercept = 4; positive for $x \neq 2$;



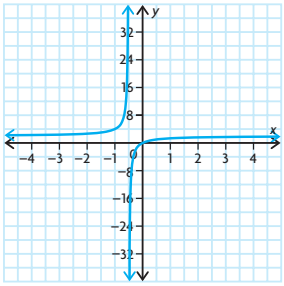
never increasing or decreasing

c) D = $\{x \in \mathbf{R} \mid x \neq 6\}$; no x-intercept; y-intercept = $\frac{1}{3}$; positive for $x \neq 6$;



never increasing or decreasing

d) $x = -0.5$; vertical asymptote: $x = -0.5$; $D = \{x \in \mathbf{R} \mid x \neq -0.5\}$; x-intercept $= 0$; y-intercept $= 0$; horizontal asymptote $= 2$; $R = \{y \in \mathbf{R} \mid x \neq 2\}$; positive on $x < -0.5$ and $x > 0$; negative on $-0.5 < x < 0$



The function is never decreasing and is increasing on $(-\infty, -0.5)$ and $(-0.5, \infty)$.

6. Answers may vary. For example, consider the function $f(x) = \frac{1}{x - 6}$. You know that the vertical asymptote would be $x = 6$. If you were to find the value of the function very close to $x = 6$ (say $f(5.99)$ or $f(6.01)$) you would be able to determine the behaviour of the function on either side of the asymptote.

$$f(5.99) = \frac{1}{(5.99) - 6} = -100$$

$$f(6.01) = \frac{1}{(6.01) - 6} = 100$$

To the left of the vertical asymptote, the function moves toward $-\infty$. To the right of the vertical asymptote, the function moves toward ∞.

7. **a)** $x = 6$

b) $x = 0.2$ and $x = -\frac{2}{3}$

c) $x = -6$ or $x = 2$

d) $x = -1$ and $x = 3$

8. about 12 min

9. $x = 1.82$ days and 3.297 days

10. **a)** $x < -3$ and $-2.873 < x < 4.873$

b) $-16 < x < -11$ and $-5 < x$

c) $-2 < x < -1.33$ and $-1 < x < 0$

d) $0 < x < 1.5$

11. $-0.7261 < t < 0$ and $t > 64.73$

12. **a)** -6; $x = 3$

b) 0.2; $x = -2$ and $x = -1$

13. **a)** 0.455 mg/L/h

b) -0.04 mg/L/h

c) The concentration of the drug in the blood stream appears to be increasing most rapidly during the first hour and a half; the graph is steep and increasing during this time.

14. $x = 5$ and $x = 8$; $x = 6.5$

15. **a)** As the x-coordinate approaches the vertical asymptote of a rational function, the line tangent to graph will get closer and closer to being a vertical line. This means that the slope of the line tangent to the graph will get larger and larger, approaching positive or negative infinity depending on the function, as x gets closer to the vertical asymptote.

b) As the x-coordinate grows larger and larger in either direction, the line tangent to the graph will get closer and closer to being a horizontal line. This means that the slope of the line tangent to the graph will always approach zero as x gets larger and larger.

Chapter Self-Test, p. 310

1. **a)** B

b) A

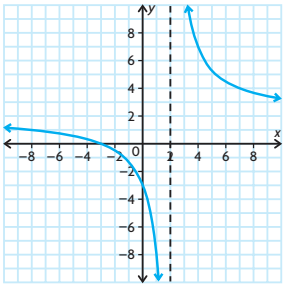
2. **a)** If $f(n)$ is very large, then that would make $\frac{1}{f(n)}$ a very small fraction.

b) If $f(n)$ is very small (less than 1), then that would make $\frac{1}{f(n)}$ very large.

c) If $f(n) = 0$, then that would make $\frac{1}{f(n)}$ undefined at that point because you cannot divide by 0.

d) If $f(n)$ is positive, then that would make $\frac{1}{f(n)}$ also positive because you are dividing two positive numbers.

3.



4. 4326 kg; \$0.52/kg

5. **a)** Algebraic; $x = -1$ and $x = -3$

b) Algebraic with factor table

The inequality is true on $(-10, -5.5)$ and on $(-5, 1.2)$.

6. **a)** To find the vertical asymptotes of the function, find the zeros of the expression in the denominator. To find the equation of the horizontal asymptotes, divide the first two terms of the expressions in the numerator and denominator.

b) This type of function will have a hole when both the numerator and the denominator share the same factor $(x + a)$.

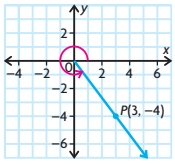
Chapter 6

Getting Started, p. 314

1. **a)** 28°

b) 332°

2. **a)**



$\sin\theta = -\frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = -\frac{4}{3}$, $\csc\theta = -\frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

b) 307°

3. **a)** $\frac{\sqrt{3}}{2}$ **c)** $\frac{\sqrt{3}}{2}$ **e)** $-\sqrt{2}$

b) 0 **d)** $\frac{1}{2}$ **f)** -1

4. **a)** 60°, 300°

b) 30°, 210°

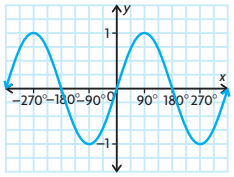
c) 45°, 225°

d) 180°

e) 135°, 315°

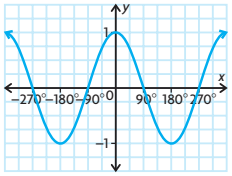
f) 90°

5. **a)**



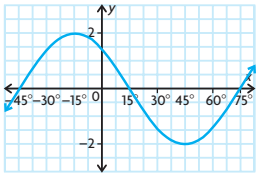
period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

b)

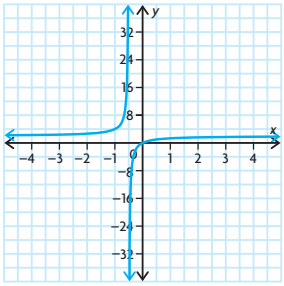


period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

6. **a)** period $= 120°$; $y = 0$; 45° to the left; amplitude $= 2$



d) $x = -0.5$; vertical asymptote: $x = -0.5$; $D = \{x \in \mathbf{R} \mid x \neq -0.5\}$; x-intercept $= 0$; y-intercept $= 0$; horizontal asymptote $= 2$; $R = \{y \in \mathbf{R} \mid x \neq 2\}$; positive on $x < -0.5$ and $x > 0$; negative on $-0.5 < x < 0$



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Chapter Self-Test, p. 310

1. **a)** B

b) A

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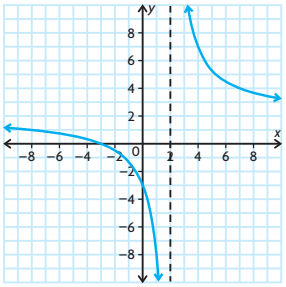
3.



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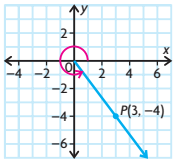
Chapter 6

Getting Started, p. 314

1. **a)** 28°

b) 332°

2. **a)**



$\sin\theta = -\frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = -\frac{4}{3}$, $\csc\theta = -\frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = -\frac{3}{4}$

b) 307°

3. **a)** $\frac{\sqrt{3}}{2}$ **c)** $\frac{\sqrt{3}}{2}$ **e)** $-\sqrt{2}$

b) 0 **d)** $\frac{1}{2}$ **f)** -1

4. **a)** 60°, 300°

b) 30°, 210°

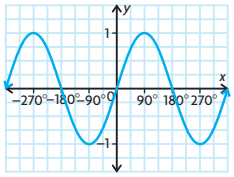
c) 45°, 225°

d) 180°

e) 135°, 315°

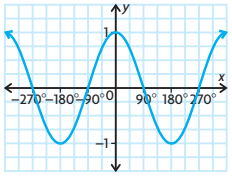
f) 90°

5. **a)**



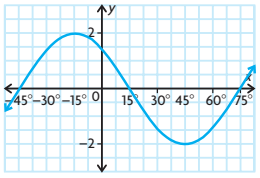
period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

b)

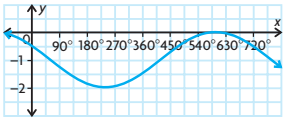


period $= 360°$; amplitude $= 1$; $y = 0$; $R = \{y \in \mathbf{R} \mid -1 \leq y \leq 1\}$

6. **a)** period $= 120°$; $y = 0$; 45° to the left; amplitude $= 2$



b) period = 720°; $y = -1$; 60° to the right; amplitude = 1



7. a is the amplitude, which determines how far above and below the axis of the curve of the function rises and falls; k defines the period of the function, which is how often the function repeats itself; d is the horizontal shift, which shifts the function to the right or the left; and c is the vertical shift of the function.

Lesson 6.1, pp. 320–322

1. **a)** π radians; 180°

b) $\frac{\pi}{2}$ radians; 90°

c) $-\pi$ radians; $-180°$

d) $-\frac{3\pi}{2}$ radians; $-270°$

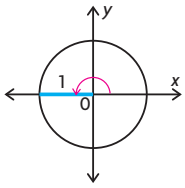
e) -2π radians; $-360°$

f) $\frac{3\pi}{2}$ radians; 270°

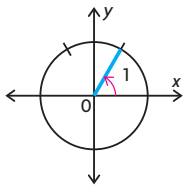
g) $-\frac{4\pi}{3}$ radians $= -240°$

h) $\frac{2\pi}{3}$ radians; 120°

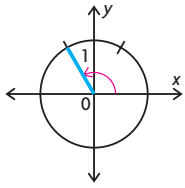
2. **a)**



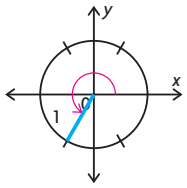
b)



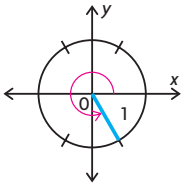
c)



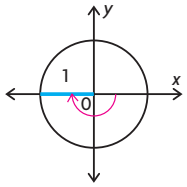
d)



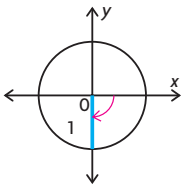
e)



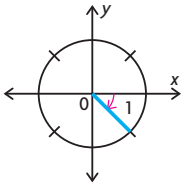
f)



g)



h)



3. **a)** $\frac{5\pi}{12}$ radians **b)** $\frac{10\pi}{9}$ radians **c)** $\frac{20\pi}{9}$ radians **d)** $= \frac{16\pi}{9}$ radians

4. **a)** 300° **b)** 54° **c)** 171.89° **d)** 495°

5. **a)** 2 radians; 114.6°

b) $\frac{25\pi}{9}$ cm

6. **a)** 28 cm

b) $\frac{40\pi}{3}$ cm

7. **a)** $\frac{\pi}{2}$ radians **b)** $\frac{3\pi}{2}$ radians **c)** π radians **d)** $\frac{\pi}{4}$ radians **e)** $\frac{5\pi}{4}$ radians **f)** $\frac{\pi}{3}$ radians **g)** $\frac{4\pi}{3}$ radians **h)** $\frac{4\pi}{3}$ radians

8. **a)** 120° **b)** 60° **c)** 45° **d)** 225° **e)** 210° **f)** 90° **g)** 330° **h)** 270°

9. **a)** $\frac{247\pi}{4}$ m

b) 162.5 m

c) $\frac{325\pi}{6}$ cm

10. $4.50\sqrt{2}$ cm

11. **a)** $\doteq 0.418\,88$ radians/s

b) $\doteq 377.0$ m

12. **a)** 36

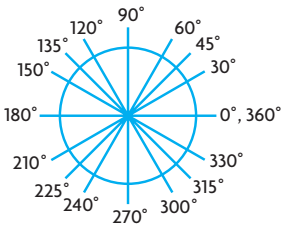
b) 0.8 m

13. **a)** equal to

b) greater than

c) stay the same

14.



$0° = 0$ radians; $30° = \frac{\pi}{6}$ radians;

$45° = \frac{\pi}{4}$ radians; $60° = \frac{\pi}{3}$ radians;

$90° = \frac{\pi}{2}$ radians; $120° = \frac{2\pi}{3}$ radians;

$135° = \frac{3\pi}{4}$ radians; $150° = \frac{5\pi}{6}$ radians;

$180° = \pi$ radians; $210° = \frac{7\pi}{6}$ radians;

$225° = \frac{5\pi}{4}$ radians; $240° = \frac{4\pi}{3}$ radians;

$270° = \frac{3\pi}{2}$ radians; $300° = \frac{5\pi}{3}$ radians;

$315° = \frac{7\pi}{4}$ radians; $330° = \frac{11\pi}{6}$ radians;

$360° = 2\pi$ radians

15. Circle B, Circle A, and Circle C

16. about 144.5 radians/s

Lesson 6.2, pp. 330–332

1. **a)** second quadrant; $\frac{\pi}{4}$; positive

b) fourth quadrant; $\frac{\pi}{3}$; positive

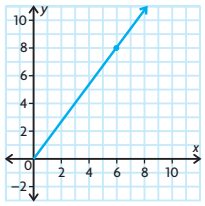
c) third quadrant; $\frac{\pi}{3}$; positive

d) second quadrant; $\frac{\pi}{6}$; negative

e) second quadrant; $\frac{\pi}{3}$; negative

f) fourth quadrant; $\frac{\pi}{4}$; negative

2. **a)** **i)**

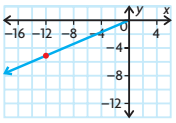


ii) $r = 10$

iii) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\csc\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$

iv) $\theta \doteq 0.93$

b) **i)**

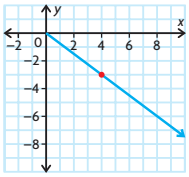


ii) $r = 13$

iii) $\sin\theta = -\frac{5}{13}$, $\cos\theta = -\frac{12}{13}$, $\tan\theta = \frac{5}{12}$, $\csc\theta = -\frac{13}{5}$, $\sec\theta = -\frac{13}{12}$, $\cot\theta = \frac{12}{5}$

iv) $\theta \doteq 3.54$

c) **i)**

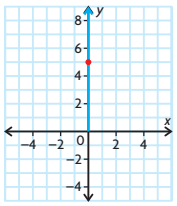


ii) $r = 5$

iii) $\sin\theta = -\frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = -\frac{3}{4}$, $\csc\theta = -\frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = -\frac{4}{3}$

iv) $\theta \doteq 5.64$

d) **i)**



ii) $r = 5$

iii) $\sin\theta = \frac{5}{5} = 1$, $\cos\theta = \frac{0}{5} = 0$, $\tan\theta = \frac{5}{0} =$ undefined, $\csc\theta = \frac{5}{5} = 1$, $\sec\theta = \frac{5}{0} =$ undefined, $\cot\theta = \frac{0}{5} = 0$

iv) $\theta \doteq \frac{\pi}{2}$

3. **a)** $\sin\left(-\frac{\pi}{2}\right) = -1$, $\cos\left(-\frac{\pi}{2}\right) = 0$, $\tan\left(-\frac{\pi}{2}\right) =$ undefined, $\csc\left(-\frac{\pi}{2}\right) = -1$, $\sec\left(-\frac{\pi}{2}\right) =$ undefined, $\cot\left(-\frac{\pi}{2}\right) = 0$

b) $\sin(-\pi) = 0$, $\cos(-\pi) = -1$, $\tan(-\pi) = 0$, $\csc(-\pi) =$ undefined, $\sec(-\pi) = -1$, $\cot(-\pi) =$ undefined

c) $\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, $\cos\left(\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$, $\tan\left(\frac{7\pi}{4}\right) = -1$, $\csc\left(\frac{7\pi}{4}\right) = -\sqrt{2}$, $\sec\left(\frac{7\pi}{4}\right) = \sqrt{2}$, $\cot\left(\frac{7\pi}{4}\right) = -1$

d) $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, $\cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, $\tan\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$, $\csc\left(-\frac{\pi}{6}\right) = -2$, $\sec\left(-\frac{\pi}{6}\right) = \frac{2\sqrt{3}}{3}$, $\cot\left(-\frac{\pi}{6}\right) = -\sqrt{3}$

4. **a)** $\sin\frac{\pi}{6}$ **b)** $\cos\frac{\pi}{3}$ **c)** $\cot\frac{3\pi}{4}$ **d)** $\sec\frac{5\pi}{6}$

5. **a)** $\frac{\sqrt{3}}{2}$ **b)** $-\frac{\sqrt{2}}{2}$ **c)** $-\frac{\sqrt{3}}{3}$ **d)** $-\frac{\sqrt{2}}{2}$ **e)** 2 **f)** 2

6. **a)** $\frac{4\pi}{3}$ **b)** $\frac{11\pi}{6}$ **c)** $\frac{5\pi}{4}$ **d)** $\frac{7\pi}{6}$ **e)** $\frac{3\pi}{2}$ **f)** π

7. **a)** $\theta \doteq 2.29$ **b)** $\theta \doteq 0.17$ **c)** $\theta \doteq 1.30$ **d)** $\theta \doteq 3.61$ **e)** $\theta \doteq 0.84$ **f)** $\theta \doteq 6.12$

8. **a)** $\cos\frac{5\pi}{4}$ **b)** $\tan\frac{5\pi}{6}$ **c)** $\csc\frac{4\pi}{3}$ **d)** $\cot\frac{5\pi}{3}$ **e)** $\sin\frac{7\pi}{6}$ **f)** $\sec\frac{\pi}{4}$

9. $\pi - 0.748 \doteq 2.39$

10. $x \doteq 5.55$ cm

11. $x \doteq 4.5$ cm

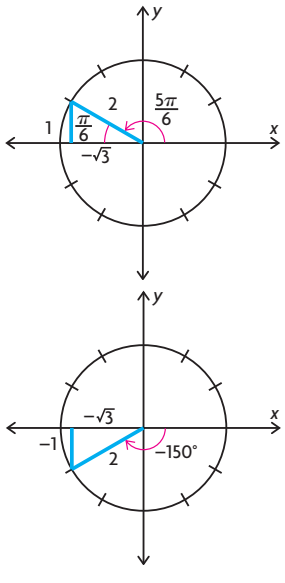
12. Draw the angle and determine the measure of the reference angle. Use the CAST rule to determine the sign of each of the ratios in the quadrant in which the angle terminates. Use this sign and the value of the ratios of the reference angle to determine the values of the primary trigonometric ratios for the given angle.

13. **a)** second or third quadrant

b) $\sin\theta = \frac{12}{13}$ or $-\frac{12}{13}$, $\tan\theta = \frac{12}{5}$ or $-\frac{12}{5}$, $\sec\theta = -\frac{13}{5}$, $\csc\theta = \frac{13}{12}$ or $-\frac{13}{12}$, $\cot\theta = \frac{5}{12}$ or $-\frac{5}{12}$

c) $\theta \doteq 1.97$ or 4.32

14.



By examining the special triangles, we see

$$\cos\left(\frac{5\pi}{6}\right) = \cos(-150°) = -\frac{\sqrt{3}}{2}$$

15.
$$2\left(\sin^2\left(\frac{11\pi}{6}\right)\right) - 1 = 2\left(-\frac{1}{2}\right)^2 - 1$$
$$= 2\left(\frac{1}{4}\right) - 1$$
$$= -\frac{1}{2}$$

$$\left(\sin^2\frac{11\pi}{6}\right) - \left(\cos^2\frac{11\pi}{6}\right)$$
$$= \left(-\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1}{4} - \frac{3}{4}$$
$$= -\frac{1}{2}$$

$$2\left(\sin^2\left(\frac{11\pi}{6}\right)\right) - 1 = \left(\sin^2\frac{11\pi}{6}\right) - \left(\cos^2\frac{11\pi}{6}\right)$$

16. $AB = 16$;

$$\sin D = \frac{8}{8\sqrt{2}} = \frac{\sqrt{2}}{2};$$

$$\cos D = \frac{8}{8\sqrt{2}} = \frac{\sqrt{2}}{2};$$

$$\tan D = \frac{8}{8} = 1$$

17. **a)** The first and second quadrants both have a positive y-value.

b) The first quadrant has a positive y-value, and the fourth quadrant has a negative y-value.

c) The first quadrant has a positive x-value, and the second quadrant has a negative x-value.

d) The first quadrant has a positive x-value and a positive y-value, and the third quadrant has a negative x-value and a negative y-value.

18. 1

19. $\cos 150° \doteq -0.26$

20. The ranges of the cosecant and secant functions are both $\{y \in \mathbf{R} \mid -1 \geq y \text{ or } y \geq 1\}$. In other words, the values of these functions can never be between -1 and 1. For the values of these functions to be between -1 and 1, the values of the sine and cosine functions would have to be greater than 1 and less than -1, which is never the case.

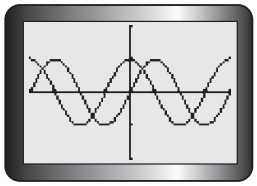
21. $\frac{2\sqrt{3} - 3}{4}$

Lesson 6.3, p. 336

1. **a)** $y = \sin\theta$ and $y = \cos\theta$ have the same period, axis, amplitude, maximum value, minimum value, domain, and range. They have different y- and θ-intercepts.

b) $y = \sin\theta$ and $y = \tan\theta$ have no characteristics in common except for their y-intercept and zeros.

2. **a)**



b) $\theta = -5.50, \theta = -2.36, \theta = 0.79, \theta = 3.93$

c) **i)** $t_n = n\pi, n \in \mathbf{I}$

ii) $t_n = \frac{\pi}{2} + 2n\pi, n \in \mathbf{I}$

iii) $t_n = \frac{3\pi}{2} + 2n\pi, n \in \mathbf{I}$

3. **a)** $t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$

b) $t_n = 2n\pi, n \in \mathbf{I}$

c) $t_n = -\pi + 2n\pi, n \in \mathbf{I}$

4. The two graphs appear to be identical.

5. **a)** $t_n = n\pi, n \in \mathbf{I}$

b) $t_n = \frac{\pi}{2} + n\pi, n \in \mathbf{I}$

Lesson 6.4, pp. 343–346

1. **a)** period: $\frac{\pi}{2}$
amplitude: 0.5
horizontal translation: 0
equation of the axis: $y = 0$

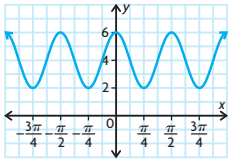
b) period: 2π
amplitude: 1
horizontal translation: $\frac{\pi}{4}$
equation of the axis: $y = 3$

c) period: $\frac{2\pi}{3}$
amplitude: 2
horizontal translation: 0
equation of the axis: $y = -1$

d) period: π
amplitude: 5
horizontal translation: $\frac{\pi}{6}$
equation of the axis: $y = -2$

2. Only the last one is cut off.

3.



period: $\frac{\pi}{2}$
amplitude: 2
horizontal translation: $\frac{\pi}{4}$ to the left
equation of the axis: $y = 4$

4. **a)** $f(x) = 25\sin(2x) - 4$

b) $f(x) = \frac{2}{5}\sin\left(\frac{\pi}{5}x\right) + \frac{1}{15}$

c) $f(x) = 80\sin\left(\frac{1}{3}x\right) - \frac{9}{10}$

d) $f(x) = 11\sin(4\pi x)$

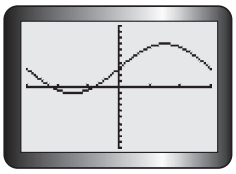
5. **a)** period $= 2\pi$, amplitude $= 18$, equation of the axis is $y = 0$; $y = 18\sin x$

b) period $= 4\pi$, amplitude $= 6$, equation of the axis is $y = -2$; $y = -6\sin(0.5x) - 2$

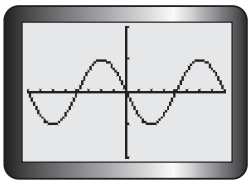
c) period $= 6\pi$, amplitude $= 2.5$, equation of the axis is $y = 6.5$; $y = -2.5\cos\left(\frac{1}{3}x\right) + 6.5$

d) period $= 4\pi$, amplitude $= 2$, equation of the axis is $y = -1$; $y = -2\cos\left(\frac{1}{2}x\right) - 1$

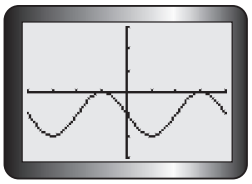
6. **a)** vertical stretch by a factor of 4, vertical translation 3 units up



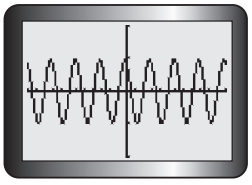
b) reflection in the x–axis, horizontal stretch by a factor of 4



c) horizontal translation π to the right, vertical translation 1 unit down



d) horizontal compression by a factor of $\frac{1}{4}$, horizontal translation $\frac{\pi}{6}$ to the left



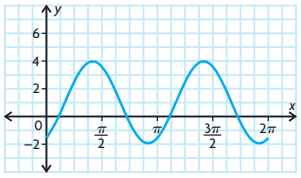
7. **a)** $f(x) = \frac{1}{2}\cos x + 3$

b) $f(x) = \cos\left(-\frac{1}{2}x\right)$

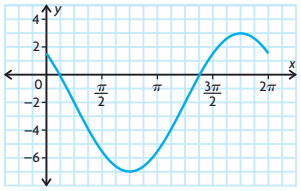
c) $f(x) = 3\cos\left(x - \frac{\pi}{2}\right)$

d) $f(x) = \cos\left(2\left(x + \frac{\pi}{2}\right)\right)$

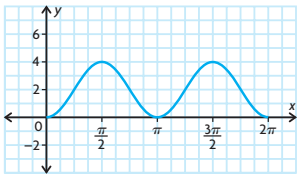
8. **a)**



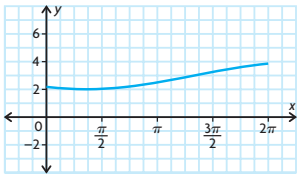
b)



c)



d)



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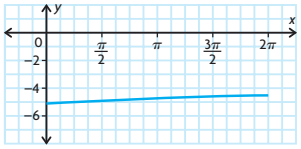
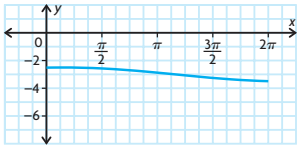


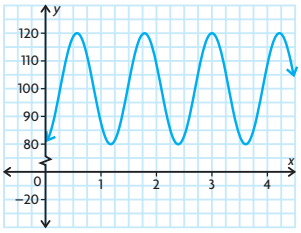
f)



9. **a)** The period of the function is $\frac{6}{5}$. This represents the time between one beat of a person's heart and the next beat.

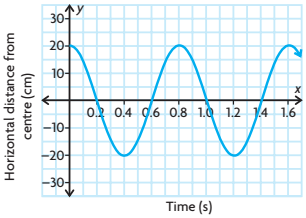
b) 80

c)



d) The range for the function is between 80 and 120. The range means the lowest blood pressure is 80 and the highest blood pressure is 120.

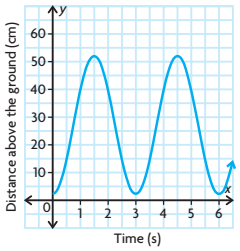
10. **a)**



b) There is a vertical stretch by a factor of 20, followed by a horizontal compression by a of factor of $\frac{2}{5\pi}$, and then a horizontal translation 0.2 to the left.

c) $y = 20\sin\left(\frac{5\pi}{2}(x + 0.2)\right)$

11. **a)**



b) vertical stretch by a factor of 25, reflection in the x-axis, vertical translation 27 units up, horizontal compression by a factor of $\frac{1}{|k|} = \frac{3}{2\pi}$

c) $y = -25\cos\left(\frac{2\pi}{3}x\right) + 27$

12. $\frac{2\pi}{7}$

13. Answers may vary. For example, $\left(\frac{14\pi}{13}, 5\right)$.

14. **a)** $y = \cos(4\pi x)$

b) $y = -2\sin\left(\frac{\pi}{4}x\right)$

c) $y = 4\sin\left(\frac{\pi}{20}(x - 10)\right) - 1$

15.

Start with graph of $y = \sin x$.

↓

Reflect in the x-axis and stretch vertically by a factor of 2 to produce graph of $y = -2\sin x$.

↓

Stretch horizontally by a factor of 2 to produce graph of $y = -2\sin(0.5x)$.

↓

Translate $\frac{\pi}{4}$ units to the right to produce graph of $y = -2\sin\left(0.5\left(x - \frac{\pi}{4}\right)\right)$.

↓

Translate 3 units up to produce graph of $y = -2\sin\left(0.5\left(x - \frac{\pi}{4}\right)\right) + 3$.

16. **a)** 100 m

b) 400 m

c) 300 m

d) 80 s

e) about 23.561 94 m/s

Mid-Chapter Review, p. 349

1. **a)** 22.5°

b) 720°

c) 286.5°

d) 165°

2. **a)** 125° ≐ 2.2 radians

b) 450° ≐ 7.9 radians

c) 5° ≐ 0.1 radians

d) 330° ≐ 5.8 radians

e) 215° ≐ 3.8 radians

f) −140° ≐ −2.4 radians

3. **a)** 20π

b) 4π radians/s

c) 380π cm

4. **a)** $\frac{\sqrt{2}}{2}$

b) $-\frac{1}{2}$

c) $-\sqrt{3}$

d) $-\frac{\sqrt{3}}{3}$

e) 0

f) $-\frac{1}{2}$

5. **a)** about 1.78

b) about 0.86

c) about 1.46

d) about 4.44

e) about 0.98

f) about 4.91

6. **a)** $\sin\dfrac{\pi}{6}$

b) $\cot\dfrac{3\pi}{4}$

c) $\sec\dfrac{\pi}{2}$

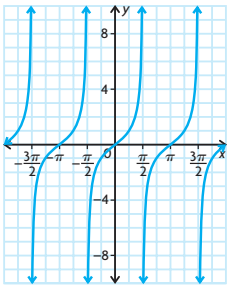
d) $\cos\dfrac{5\pi}{6}$

7. **a)** $x = 0, \pm\pi, \pm 2\pi, \ldots ; y = 0$

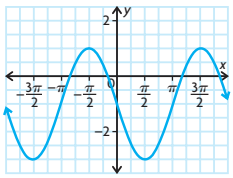
b) $x = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}, \ldots ; y = 1$

c) $x = 0, \pm\pi, \pm 2\pi, \ldots ; y = 0$

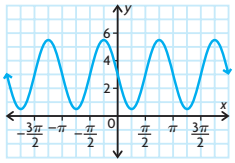
8. **a)**



b)



c)



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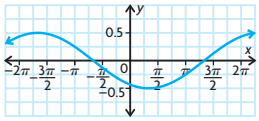
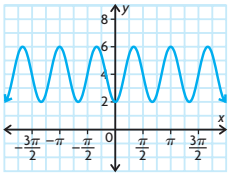
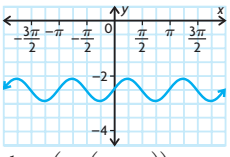


e)



f)



9. $y = \dfrac{1}{3}\sin\left(-3\left(x + \dfrac{\pi}{8}\right)\right) - 23$

Lesson 6.5, p. 353

1. **a)** $t_n = n\pi, n \in \mathbf{I}$

b) no maximum value

c) no minimum value

2. **a)** $t_n = \dfrac{\pi}{2} + n\pi, n \in \mathbf{I}$

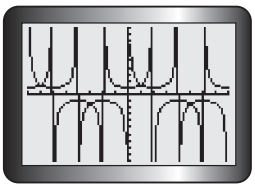
b) no maximum value

c) no minimum value

3. **a)** $t_n = n\pi, n \in \mathbf{I}$

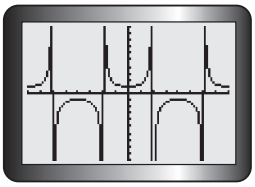
b) $t_n = \dfrac{\pi}{2} + n\pi, n \in \mathbf{I}$

4.



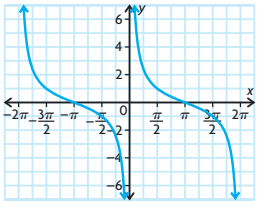
$-5.50, -2.35, 0.79, 3.93$

5. Yes, the graphs of $y = \csc\left(x + \dfrac{\pi}{2}\right)$ and $y = \sec x$ are identical.

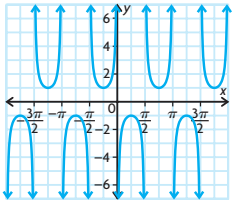


6. Answers may vary. For example, reflect the graph of $y = \tan x$ across the y-axis and then translate the graph $\dfrac{\pi}{2}$ units to the left.

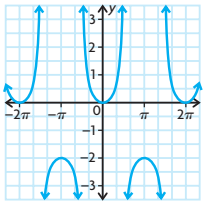
7. **a)** period $= 2\pi$



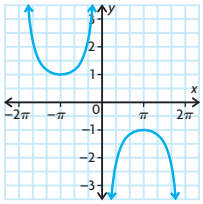
b) period $= \pi$



c) period $= 2\pi$



d) period $= 4\pi$

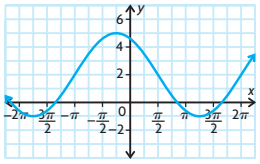


Lesson 6.6, pp. 360–362

1. $y = 3\cos\left(\dfrac{2}{3}\left(x + \dfrac{\pi}{4}\right)\right) + 2$

2. 2, 0.5, $y \doteq 0.973\,94$

3.



$x = 1.3$

4. amplitude and equation of the axis

5. **a)** the radius of the circle in which the tip of the sparkler is moving

b) the time it takes Mike to make one complete circle with the sparkler

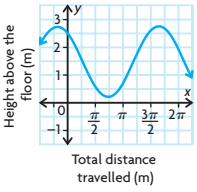
c) the height above the ground of the centre of the circle in which the tip of the sparkler is moving

d) cosine function

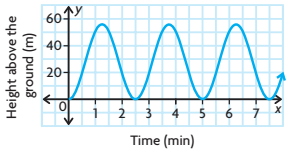
6. $y = 90\sin\left(\dfrac{\pi}{12}x\right) + 30$

7. $y = 250\cos\left(\dfrac{2\pi}{3}x\right) + 750$

8. $y = -1.25\sin\left(\dfrac{4}{5}x\right) + 1.5$



9. 0.98 min $< t <$ 1.52 min,
3.48 min $< t <$ 4.02 min,
5.98 min $< t <$ 6.52 min



10. **a)** $y = 3.7 \sin\left(\frac{2\pi}{365}x\right) + 12$

b) $y \doteq 13.87$ hours

11. $T(t) = 16.2 \sin\left(\frac{2\pi}{365}(t - 116)\right) + 1.4$,
$0 < t < 111$ and $304 < t < 365$

12. The student should graph the height of the nail above the ground as a function of the total distance travelled by the nail, because the nail would not be travelling at a constant speed. If the student graphed the height of the nail above the ground as a function of time, the graph would not be sinusoidal.

13. minute hand:
$D(t) = 15 \cos\left(\frac{\pi}{30}t\right) + 300$;
second hand:
$D(t) = 15 \cos(2\pi t) + 300$;
hour hand:
$D(t) = 8 \cos\left(\frac{\pi}{360}t\right) + 300$

Lesson 6.7, pp. 369–373

1. **a)** $0 < x < \pi, \pi < x < 2\pi$

b) $-\frac{\pi}{2} < x < \frac{\pi}{2}, \frac{3\pi}{2} < x < \frac{5\pi}{2}$

c) $\frac{\pi}{2} < x < \frac{3\pi}{2}, \frac{5\pi}{2} < x < 3\pi$

2. **a)** $x = \frac{\pi}{4}, x = \frac{5\pi}{4}$

b) $x = \frac{\pi}{2}, x = \frac{5\pi}{2}$

c) $x = 0, x = 2\pi$

3. 0

4. **a)** about 0.465

b) 0

c) about −0.5157

d) about −1.554

5. **a)** $0 < x < \frac{\pi}{2}, \pi < x < \frac{3\pi}{2}$

b) $0 < x < \frac{\pi}{4}, \pi < x < \frac{5\pi}{4}$

c) $\frac{\pi}{4} < x < \frac{\pi}{2}, \frac{5\pi}{4} < x < \frac{3\pi}{2}$

6. **a)** $x = \frac{1}{4}, x = \frac{3}{4}$

b) $x = 0, x = 1$

c) $x = \frac{1}{2}, x = \frac{3}{2}$

7. **a)** about −0.7459

b) about −1.310

c) 0

8. negative

9. **a)** $R(t) = 4.5 \cos\left(\frac{\pi}{12}t\right) + 20.2$

b) fastest: $t = 6$ months, $t = 18$ months, $t = 30$ months, $t = 42$ months;
slowest: $t = 0$ months, $t = 12$ months, $t = 24$ months, $t = 36$ months, $t = 48$ months

c) about 1.164 mice per owl/s

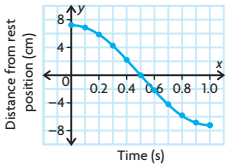
10. **a)** **i)** 0.25 t/h

ii) about 0.2588 t/h

iii) 0.2612 t/h

b) The estimate calculated in part iii) is the most accurate. The smaller the interval, the more accurate the estimate.

11. **a)**



b) half of one cycle

c) −14.4 cm/s

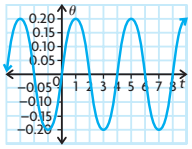
d) The bob is moving the fastest when it passes through its rest position. You can tell because the images of the balls are farthest apart at this point.

e) The pendulum's rest position is halfway between the maximum and minimum values on the graph. Therefore, at this point, the pendulum's instantaneous rate of change is at its maximum.

12. **a)** 0

b) −0.5 m/s

13. **a)**



b) 0.2 radians/s

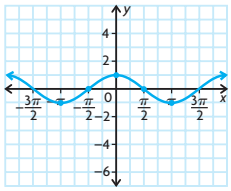
c) Answers may vary. For example, about $-\frac{2}{3}$ radians/s.

d) $t = 0, 2, 4, 6$, and 8

14. Answers may vary. For example, for $x = 0$, the instantaneous rate of change of $f(x) = \sin x$ is approximately 0.9003, while the instantaneous rate of change of $f(x) = 3 \sin x$ is approximately 2.7009. (The interval $-\frac{\pi}{4} < x < \frac{\pi}{4}$ was used.) Therefore, the instantaneous rate of change of $f(x) = 3 \sin x$ is at its maximum three times more than the instantaneous rate of change of $f(x) = \sin x$. However, there are points where the instantaneous rate of change is the same for the two functions. For example, at $x = \frac{\pi}{2}$, it is 0 for both functions.

15. **a)** −1, 0, 1, 0, and −1

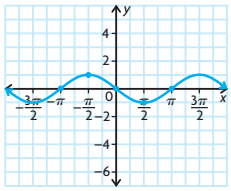
b)



The function is $f(x) = \cos x$. Based on this information, the derivative of $f(x) = \sin x$ is $\cos x$.

16. **a)** 0, 1, 0, −1, and 0

b)



The function is $f(x) = -\sin x$. Based on this information, the derivative of $f(x) = \cos x$ is $-\sin x$.

Chapter Review, pp. 376–377

1. $\frac{33}{16}$

2. 70π

3. **a)** $\frac{\pi}{9}$ radians

b) $\frac{-5\pi}{18}$ radians

c) $\frac{8\pi}{9}$ radians

d) $\frac{7\pi}{3}$ radians

4. **a)** 45°

b) −225°

c) 480°

d) −120°

5. **a)** $\frac{5\pi}{6}$

b) $\frac{4\pi}{3}$

c) $\frac{3\pi}{4}$

d) $\frac{7\pi}{6}$

6. **a)** $\tan\theta = \frac{12}{13}$

b) $\sec\theta = -\frac{13}{5}$

c) about 5.14

7. 2.00

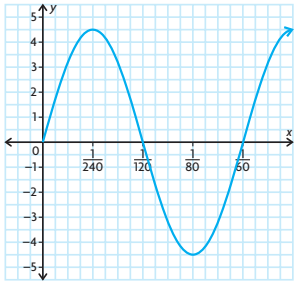
8. **a)** 2π radians
b) 2π radians
c) π radians

9. $y = 5\sin\left(x + \frac{\pi}{3}\right) + 2$

10. $y = -3\cos\left(2\left(x + \frac{\pi}{4}\right)\right) - 1$

11. **a)** reflection in the x-axis, vertical stretch by a factor of 19, vertical translation 9 units down
b) horizontal compression by a factor of $\frac{1}{10}$, horizontal translation $\frac{\pi}{12}$ to the left
c) vertical compression by a factor of $\frac{10}{11}$, horizontal translation $\frac{\pi}{9}$ to the right, vertical translation 3 units up
d) reflection in the x-axis, reflection in the y-axis, horizontal translation π to the right

12. **a)**



b) $\frac{1}{60}$
c) $\frac{1}{240}$
d) $\frac{1}{80}$

13. **a)** 2π radians
b) 2π radians
c) π radians

14. **a)** the radius of the circle in which the bumblebee is flying
b) the time that the bumblebee takes to fly one complete circle
c) the height, above the ground, of the centre of the circle in which the bumblebee is flying
d) cosine function

15. $P(m) = 7250\cos\left(\frac{\pi}{6}m\right) + 7750$

16. $h(t) = 30\sin\left(\frac{5\pi}{3}t - \frac{\pi}{2}\right) + 150$

17. **a)** $0 < x < 5\pi$, $10\pi < x < 15\pi$
b) $2.5\pi < x < 7.5\pi$, $12.5\pi < x < 17.5\pi$
c) $0 < x < 2.5\pi$, $7.5\pi < x < 12.5\pi$

18. **a)** $x = 0, x = \frac{1}{2}$
b) $x = \frac{1}{8}, x = \frac{5}{8}$
c) $x = \frac{3}{8}, x = \frac{7}{8}$

19. **a)** $x = \frac{3}{4}$ s
b) the time between one beat of a person's heart and the next beat
c) 140
d) -129

Chapter Self-Test, p. 378

1. $y = \sec x$
2. $\sec 2\pi$
3. $y \doteq 108.5$
4. about 0.31 °C per day
5. $\frac{3\pi}{5}$, 110°, $\frac{5\pi}{8}$, 113°, and $\frac{2\pi}{3}$
6. $y = \sin\left(x + \frac{5\pi}{8}\right)$
7. $y \doteq -30$
8. **a)** $-3\cos\left(\frac{\pi}{12}x\right) + 22$
b) about 0.5 °C per hour
c) about 0 °C per hour

Cumulative Review Chapters 4–6, pp. 380–383

1.	(d)	**9.**	(c)	**17.**	(d)	**25.**	(b)
2.	(b)	**10.**	(c)	**18.**	(b)	**26.**	(d)
3.	(a)	**11.**	(d)	**19.**	(b)	**27.**	(a)
4.	(c)	**12.**	(a)	**20.**	(b)	**28.**	(c)
5.	(a)	**13.**	(d)	**21.**	(d)	**29.**	(b)
6.	(b)	**14.**	(c)	**22.**	(c)		
7.	(a)	**15.**	(d)	**23.**	(a)		
8.	(c)	**16.**	(a)	**24.**	(d)		

30. **a)** If x is the length in centimetres of a side of one of the corners that have been cut out, the volume of the box is $(50 - 2x)(40 - 2x)x$ cm^3.
b) 5 cm or 10 cm
c) $x \doteq 7.4$ cm
d) $3 < x < 12.8$

31. **a)** The zeros of $f(x)$ are $x = 2$ or $x = 3$. The zero of $g(x)$ is $x = 3$. The zero of $\frac{f(x)}{g(x)}$ is $x = 2$. $\frac{g(x)}{f(x)}$ does not have any zeros.
b) $\frac{f(x)}{g(x)}$ has a hole at $x = 3$; no asymptotes. $\frac{g(x)}{f(x)}$ has an asymptote at $x = 2$ and $y = 0$.
c) $x = 1; \frac{f(x)}{g(x)}: y = x - 2, \frac{g(x)}{f(x)}: y = -x$

32. **a)** Vertical compressions and stretches do not affect location of zeros; maximum and minimum values are multiplied by the scale factor, but locations are unchanged; instantaneous rates of change are multiplied by the scale factor.
Horizontal compressions and stretches move locations of zeros, maximums, and minimums toward or away from the y-axis by the reciprocal of the scale factor; instantaneous rates of change are multiplied by the reciprocal of scale factor.
Vertical translations change location of zeros or remove them; maximum and minimum values are increased or decreased by the amount of the translation, but locations are unchanged; instantaneous rates of change are unchanged.
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b) For $y = \cos x$, the answer is the same as in part **a)**, except that a horizontal reflection does not affect instantaneous rates of change. For $y = \tan x$, the answer is also the same as in part **a)**, except that nothing affects the maximum and minimum values, since there are no maximum or minimum values for $y = \tan x$.

Chapter 7

Getting Started, p. 386

1. **a)** 1
b) $-\frac{22}{7}$
c) 8 or -3
d) $\frac{2}{3}$ or $-\frac{5}{2}$
e) $-1 \pm \sqrt{2}$
f) $\frac{3 \pm \sqrt{21}}{6}$

2. To do this, you must show that the two distances are equal:

$$D_{AB} = \sqrt{(2-1)^2 + \left(\frac{1}{2} - 0\right)^2} = \frac{\sqrt{5}}{2};$$

$$D_{CD} = \sqrt{\left(0 - \frac{1}{2}\right)^2 + (6-5)^2} = \frac{\sqrt{5}}{2}.$$

Since the distances are equal, the line segments are the same length.

3. **a)** $\sin A = \frac{8}{17}$, $\cos A = \frac{15}{17}$, $\tan A = \frac{8}{15}$, $\csc A = \frac{17}{8}$, $\sec A = \frac{17}{15}$, $\cot A = \frac{15}{8}$
b) 0.5 radians
c) 61.9°

8. **a)** 2π radians
b) 2π radians
c) π radians

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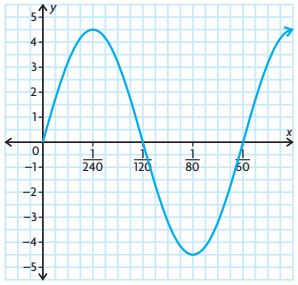

b) $\frac{1}{60}$
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Cumulative Review Chapters 4–6, pp. 380–383

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Chapter 7

Getting Started, p. 386

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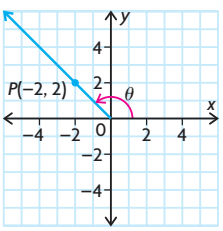
4. **a)**



b) $\frac{\pi}{4}$ radians **c)** $\frac{3\pi}{4}$ radians

5. **a)** $A: \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$; $F: \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$;
$B: \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $G: \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$;
$C: \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$; $H: \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$;
$D: \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$; $I: \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$;
$E: \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$; $J: \left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

b) **i)** $-\frac{\sqrt{2}}{2}$ **ii)** $-\frac{1}{2}$ **iii)** -1 **iv)** 2

6. **a)** If the angle x is in the second quadrant:
$\sin x = \frac{3}{5}$; $\cos x = -\frac{4}{5}$;
$\csc x = \frac{5}{3}$; $\sec x = -\frac{5}{4}$; $\cot x = -\frac{4}{3}$.
If the angle x is in the fourth quadrant:
$\sin x = -\frac{3}{5}$; $\cos x = \frac{4}{5}$; $\csc x = -\frac{5}{3}$;
$\sec x = \frac{5}{4}$; $\cot x = -\frac{4}{3}$

b) If x is in the second quadrant, $x = 2.5$.
If x is in the fourth quadrant, $x = 5.6$.

7. **a)** true **d)** false
b) true **e)** true
c) false **f)** true

8.

Perform a vertical stretch/compression by a factor of $|a|$.

↓

Use $\left|\frac{1}{k}\right|$ to determine the horizontal stretch/compression.

↓

Use a and k to determine whether the function is reflected in the y-axis or the x-axis.

↓

Perform a vertical translation of c units up or down.

↓

Perform a horizontal translation of d units to the right or the left.

Lesson 7.1, pp. 392–393

1. **a)** Answers may vary. For example:
$y = \cos(\theta + 2\pi)$, $y = \cos(\theta + 4\pi)$, $y = \cos(\theta - 2\pi)$

b) $y = \sin\left(\theta + \frac{\pi}{2}\right)$, $y = \sin\left(\theta - \frac{3\pi}{2}\right)$, $y = \sin\left(\theta + \frac{5\pi}{2}\right)$

2. **a)** $y = \csc\theta$ is odd, $\csc(-\theta) = -\csc\theta$;
$y = \sec\theta$ is even, $\sec(-\theta) = \sec\theta$;
$y = \cot\theta$ is odd, $\cot(-\theta) = -\cot\theta$

b) $y = \cot(-\theta)$ is the graph of $y = \cot\theta$ reflected across the y-axis; $y = -\cot\theta$ is the graph of $y = \cot\theta$ reflected across the x-axis. Both of these transformations result in the same graph. $y = \csc(-\theta)$ is the graph of $y = \csc\theta$ reflected across the y-axis; $y = -\csc\theta$ is the graph of $y = \csc\theta$ reflected across the x-axis. Both of these transformations result in the same graph. $y = \sec(-\theta)$ is the graph of $y = \sec\theta$ reflected across the y-axis. This results in the same graph as $y = \sec\theta$.

3. **a)** $\cos\frac{\pi}{3}$ **c)** $\cot\frac{\pi}{8}$ **e)** $\cos\frac{3\pi}{8}$
b) $\sin\frac{\pi}{12}$ **d)** $\sin\frac{3\pi}{16}$ **f)** $\cot\frac{\pi}{3}$

4. **a)** $\csc\theta = \sec\left(\frac{\pi}{2} - \theta\right)$;
$\sec\theta = \csc\left(\frac{\pi}{2} - \theta\right)$;
$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right)$

b) $y = \tan\left(\frac{\pi}{2} - \theta\right) = \tan\left(-\left(\theta - \frac{\pi}{2}\right)\right)$;
This is the graph of $y = \tan\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \cot\theta$.
$y = \csc\left(\frac{\pi}{2} - \theta\right) = \csc\left(-\left(\theta - \frac{\pi}{2}\right)\right)$;
This is the graph of $y = \csc\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \sec\theta$.
$y = \sec\left(\frac{\pi}{2} - \theta\right) = \sec\left(-\left(\theta - \frac{\pi}{2}\right)\right)$;
This is the graph of $y = \sec\theta$ reflected across the y-axis and translated $\frac{\pi}{2}$ to the right, which is identical to the graph of $y = \csc\theta$.

5. **a)** $\sin\frac{\pi}{8}$ **d)** $\cos\frac{\pi}{6}$
b) $-\cos\frac{\pi}{12}$ **e)** $-\sin\frac{3\pi}{8}$
c) $\tan\frac{\pi}{4}$ **f)** $-\tan\frac{\pi}{3}$

6. **a)** Assume the circle is a unit circle. Let the coordinates of Q be (x, y). Since P and Q are reflections of each other in the line $y = x$, the coordinates of P are (y, x). Draw a line from P to the positive x-axis. The hypotenuse of the new right triangle makes an angle of $\left(\frac{\pi}{2} - \theta\right)$ with the positive x-axis. Since the x-coordinate of P is y, $\cos\left(\frac{\pi}{2} - \theta\right) = y$. Also, since the y-coordinate of Q is y, $\sin\theta = y$. Therefore, $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$.

b) Assume the circle is a unit circle. Let the coordinates of the vertex on the circle of the right triangle in the first quadrant be (x, y). Then $\sin\theta = y$, so $-\sin\theta = -y$. The point on the circle that results from rotating the vertex by $\frac{\pi}{2}$ counterclockwise about the origin has coordinates $(-y, x)$, so $\cos\left(\frac{\pi}{2} + \theta\right) = -y$. Therefore, $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$.

7. **a)** true

b) false; Answers may vary. For example: Let $\theta = \frac{\pi}{2}$. Then the left side is $\sin\frac{\pi}{2}$, or 1. The right side is $-\sin\frac{\pi}{2}$, or -1.

c) false; Answers may vary. For example: Let $\theta = \pi$. Then the left side is $\cos\pi$, or -1. The right side is $-\cos 5\pi$, or 1.

d) false; Answers may vary. For example: Let $\theta = \frac{\pi}{4}$. Then the left side is $\tan\frac{3\pi}{4}$, or $-\frac{\sqrt{2}}{2}$. The right side is $\tan\frac{\pi}{4}$, or $\frac{\sqrt{2}}{2}$.

e) false; Answers may vary. For example: Let $\theta = \pi$. Then the left side is $\cot\frac{3\pi}{4}$, or -1. The right side is $\tan\frac{\pi}{4}$, or 1.

f) false; Answers may vary. For example: Let $\theta = \frac{\pi}{2}$. Then the left side is $\sin\frac{5\pi}{2}$, or 1. The right side is $\sin\left(-\frac{\pi}{2}\right)$, or -1.

Lesson 7.2, pp. 400–401

1. **a)** $\sin 3a$ **b)** $\cos 7x$

2. **a)** $\tan 60°; \sqrt{3}$ **b)** $\cos\frac{\pi}{3}; \frac{1}{2}$

3. **a)** $30° + 45°$ **d)** $\frac{\pi}{4} - \frac{\pi}{6}$
b) $30° - 45°$ **e)** $60° + 45°$
c) $\frac{\pi}{6} - \frac{\pi}{3}$ **f)** $\frac{\pi}{2} + \frac{\pi}{3}$

4. **a)** $\frac{\sqrt{2} + \sqrt{6}}{4}$ **d)** $\frac{\sqrt{2} - \sqrt{6}}{4}$
b) $\frac{\sqrt{2} + \sqrt{6}}{4}$ **e)** $\frac{\sqrt{2} - \sqrt{6}}{4}$
c) $2 + \sqrt{3}$ **f)** $-2 + \sqrt{3}$

5. **a)** $-\frac{1}{2}$ **d)** $-\frac{1}{2}$
b) $-\frac{\sqrt{2}}{2}$ **e)** $\frac{\sqrt{3}}{3}$
c) 1 **f)** $-\frac{\sqrt{3}}{2}$

6. **a)** $-\sin x$ **d)** $\tan x$
b) $\sin x$ **e)** $-\sin x$
c) $-\sin x$ **f)** $-\tan x$

7. **a)** $\sin(\pi + x)$ is equivalent to $\sin x$ translated π to the left, which is equivalent to $-\sin x$.

b) $\cos\left(x + \frac{3\pi}{2}\right)$ is equivalent to $\cos x$ translated $\frac{3\pi}{2}$ to the left, which is equivalent to $\sin x$.

c) $\cos\left(x + \frac{\pi}{2}\right)$ is equivalent to $\cos x$ translated $\frac{\pi}{2}$ to the left, which is equivalent to $-\sin x$.

d) $\tan(x + \pi)$ is equivalent to $\tan x$ translated π to the left, which is equivalent to $\tan x$.

e) $\sin(x - \pi)$ is equivalent to $\sin x$ translated π to the right, which is equivalent to $-\sin x$.

f) $\tan(2\pi - x)$ is equivalent to $\tan(-x)$, which is equivalent to $\tan x$ reflected in the y-axis, which is equivalent to $-\tan x$.

8. **a)** $\frac{\sqrt{6} - \sqrt{2}}{4}$ **d)** $\frac{\sqrt{2} - \sqrt{6}}{4}$
b) $-2 + \sqrt{3}$ **e)** $-2 - \sqrt{3}$
c) $\frac{-\sqrt{2} - \sqrt{6}}{4}$ **f)** $-2 - \sqrt{3}$

9. **a)** $\frac{63}{65}$ **d)** $\frac{56}{65}$
b) $-\frac{16}{65}$ **e)** $-\frac{16}{63}$
c) $-\frac{33}{65}$ **f)** $-\frac{56}{33}$

10. $\frac{323}{325}; \frac{323}{36}$

11. **a)** $\cos\left(\frac{\pi}{2} - x\right)$
$= \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$
$= (0)(\cos x) + (1)(\sin x)$
$= 0 + \sin x$
$= \sin x$

b) $\sin\left(\frac{\pi}{2} - x\right)$
$= \sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x$
$= (1)(\cos x) - (0)(\sin x)$
$= \cos x - 0$
$= \cos x$

12. **a)** 0 **b)** $-\sqrt{3}\sin x$

13. $\tan f$, $\cos f \neq 0$, $\cos g \neq 0$

14.
Write $\sin a$ in terms of $\frac{y}{r}$.
↓
Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.
↓
Since $a \in \left[0, \frac{\pi}{2}\right]$, choose the positive value of x and determine $\cos a$.
↓
Write $\sin b$ in terms of $\frac{y}{r}$.
↓
Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.
↓
Since $b \in \left[0, \frac{\pi}{2}\right]$, choose the positive value of x and determine $\cos b$.
↓
Use the formula $\cos(a + b) = \cos a \cos b - \sin a \sin b$ to evaluate $\cos(a + b)$.

15. See compound angle formulas listed on p. 399.
The two sine formulas are the same, except for the operators. Remembering that the same operator is used on both the left and right sides in both equations will help you remember the formulas.
Similarly, the two cosine formulas are the same, except for the operators. Remembering that the operator on the left side is the opposite of the operator on the right side in both equations will help you remember the formulas.
The two tangent formulas are the same, except for the operators in the numerator and the denominator on the right side. Remembering that the operators in the numerator and the denominator are opposite in both equations, and that the operator in the numerator is the same as the operator on the left side, will help you remember the formulas.

16. $2\sin\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)$
$= (2)\left(\left(\sin\frac{C}{2}\right)\left(\cos\frac{D}{2}\right) + \left(\cos\frac{C}{2}\right)\left(\sin\frac{D}{2}\right)\right)\left(\left(\cos\frac{C}{2}\right)\left(\cos\frac{D}{2}\right) + \left(\sin\frac{C}{2}\right)\left(\sin\frac{D}{2}\right)\right)$
$= (2)\left(\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\cos^2\frac{D}{2}\right) + \left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\cos^2\frac{C}{2}\right) + \left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\sin^2\frac{C}{2}\right) + \left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\sin^2\frac{D}{2}\right)\right)$
$= (2)\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right)\left(\cos^2\frac{D}{2} + \sin^2\frac{D}{2}\right) + 2\left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)\left(\cos^2\frac{C}{2} + \sin^2\frac{C}{2}\right)$
$= (2)\left(\sin\frac{C}{2}\right)\left(\cos\frac{C}{2}\right) + 2\left(\sin\frac{D}{2}\right)\left(\cos\frac{D}{2}\right)$
$= \sin\left(2\left(\frac{C}{2}\right)\right) + \sin\left(2\left(\frac{D}{2}\right)\right)$
$= \sin C + \sin D$

17. $\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$

18. Let $C = x + y$ and let $D = x - y$.
$\cos C + \cos D$
$= \cos(x + y) + \cos(x - y)$
$= \cos x \cos y - \sin x \sin y + \cos x \cos y + \sin x \sin y$
$= 2\cos x \cos y$
$\frac{C + D}{2} = \frac{x + y + x - y}{2} = x$
$\frac{C - D}{2} = \frac{x + y - x + y}{2} = y$
So $\cos C + \cos D$
$= 2\cos\left(\frac{C + D}{2}\right)\cos\left(\frac{C - D}{2}\right)$

19. Let $C = x + y$ and let $D = x - y$.
$\cos C - \cos D$
$= \cos(x + y) - \cos(x - y)$
$= \cos x \cos y - \sin x \sin y - (\cos x \cos y - \sin x \sin y)$
$= -2\sin x \sin y$
$\frac{C + D}{2} = \frac{x + y + x - y}{2} = x$
$\frac{C - D}{2} = \frac{x + y - x + y}{2} = y$
So $\cos C - \cos D$
$= -2\sin\left(\frac{C + D}{2}\right)\sin\left(\frac{C - D}{2}\right)$

Lesson 7.3, pp. 407–408

1. **a)** $\sin 10x$ **d)** $\tan 8x$
b) $\cos 2\theta$ **e)** $2\sin 2\theta$
c) $\cos 6x$ **f)** $\cos\theta$

2. **a)** $\sin 90°; 1$ **d)** $\cos\frac{\pi}{6}; \frac{\sqrt{3}}{2}$
b) $\cos 60°; \frac{1}{2}$ **e)** $\cos\frac{3\pi}{4}; -\frac{\sqrt{2}}{2}$
c) $\sin\frac{\pi}{6}; \frac{1}{2}$ **f)** $\sin 120°; \frac{\sqrt{3}}{2}$

3. **a)** $2 \sin 2\theta \cos 2\theta$

b) $2 \sin^2 (1.5x) - 1$

c) $\dfrac{2 \tan (0.5x)}{1 - \tan^2 (0.5x)}$

d) $\cos^2 3\theta - \sin^2 3\theta$

e) $2 \sin (0.5x) \cos (0.5x)$

f) $\dfrac{2 \tan (2.5\theta)}{1 - \tan^2 (2.5\theta)}$

4. $\sin 2\theta = \dfrac{24}{25}$, $\cos 2\theta = -\dfrac{7}{25}$, $\tan 2\theta = -\dfrac{24}{7}$

5. $\sin 2\theta = -\dfrac{336}{625}$, $\cos 2\theta = \dfrac{527}{625}$, $\tan 2\theta = -\dfrac{336}{527}$

6. $\sin 2\theta = -\dfrac{120}{169}$, $\cos 2\theta = -\dfrac{119}{169}$, $\tan 2\theta = \dfrac{120}{119}$

7. $\sin 2\theta = -\dfrac{24}{25}$, $\cos 2\theta = \dfrac{7}{25}$, $\tan 2\theta = -\dfrac{24}{7}$

8. $a = \dfrac{1}{2}$

9. Jim can find the sine of $\frac{\pi}{8}$ by using the formula $\cos 2x = 1 - 2 \sin^2 x$ and isolating $\sin x$ on one side of the equation. When he does this, the formula becomes $\sin x = \pm \sqrt{\dfrac{1 - \cos 2x}{2}}$. The cosine of $\frac{\pi}{4}$ is $\frac{\sqrt{2}}{2}$, so $\sin \dfrac{\pi}{8} = \pm \sqrt{\dfrac{1 - \cos \frac{\pi}{4}}{2}} = \pm \dfrac{\sqrt{2 - \sqrt{2}}}{2}$.

Since $\frac{\pi}{8}$ is in the first quadrant, the sign of $\sin \frac{\pi}{8}$ is positive.

10. Marion can find the cosine of $\frac{\pi}{12}$ by using the formula $\cos 2x = 2 \cos^2 x - 1$ and isolating $\cos x$ on one side of the equation. When she does this, the formula becomes $\cos x = \pm \sqrt{\dfrac{1 + \cos 2x}{2}}$. The cosine of $\frac{\pi}{6}$ is $\frac{\sqrt{3}}{2}$, so $\cos \dfrac{\pi}{12} = \pm \sqrt{\dfrac{1 + \cos \frac{\pi}{6}}{2}} = \pm \dfrac{\sqrt{2 + \sqrt{3}}}{2}$.

Since $\frac{\pi}{12}$ is in the first quadrant, the sign of $\cos \frac{\pi}{12}$ is positive.

11. **a)** $\sin 4x$
$= (2)(2 \sin x \cos x)(\cos 2x)$
$= (2)(2 \sin x \cos x)(1 - 2 \sin^2 x)$
$= (4 \sin x \cos x)(1 - 2 \sin^2 x)$
$= 4 \sin x \cos x - 8 \sin^3 x \cos x$

b) $\sin \dfrac{2\pi}{3} = \dfrac{\sqrt{3}}{2}$

$\sin 4\left(\dfrac{2\pi}{3}\right) = 4 \sin \dfrac{2\pi}{3} \cos \dfrac{2\pi}{3} - 8 \sin^3 \dfrac{2\pi}{3} \cos \dfrac{2\pi}{3}$

$\sin \dfrac{8\pi}{3} = (4)\left(\dfrac{\sqrt{3}}{2}\right)\left(-\dfrac{1}{2}\right) - (8)\left(\dfrac{\sqrt{3}}{2}\right)^3\left(-\dfrac{1}{2}\right)$

$\sin \dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - (-4)\left(\dfrac{3\sqrt{3}}{8}\right)$

$\sin \dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - \left(-\dfrac{3\sqrt{3}}{2}\right)$

$\sin \dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} - \left(-\dfrac{6\sqrt{3}}{4}\right)$

$\sin \dfrac{8\pi}{3} = -\dfrac{4\sqrt{3}}{4} + -\dfrac{6\sqrt{3}}{4}$

$\sin \dfrac{8\pi}{3} = \dfrac{2\sqrt{3}}{4}$

$\sin \dfrac{8\pi}{3} = \dfrac{\sqrt{3}}{2}$

12. **a)** $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
$\sin 2\theta = 2 \cos \theta \sin \theta$
$\sin 3\theta = (\sin 2\theta + \theta)$
$= (2 \cos \theta \sin \theta)(\cos \theta) + (\cos^2 \theta - \sin^2 \theta)(\sin \theta)$
$= 2 \cos^2 \theta \sin \theta + \cos^2 \theta \sin \theta - \sin^3 \theta$
$= 3 \cos^2 \theta \sin \theta - \sin^3 \theta$

b) $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
$\sin 2\theta = 2 \cos \theta \sin \theta$
$\cos 3\theta = (\cos 2\theta + \theta)$
$= (\cos^2 \theta - \sin^2 \theta)(\cos \theta) - (2 \cos \theta \sin \theta)(\sin \theta)$
$= \cos^3 \theta - \cos x \sin^2 \theta - 2 \cos \theta \sin^2 \theta$
$= \cos^3 \theta - 3 \cos \theta \sin^2 \theta$

c) $\tan 2\theta = \dfrac{2 \tan \theta}{1 - \tan^2 \theta}$

$\tan 3\theta = (\tan 2\theta + \theta)$

$= \dfrac{\frac{2 \tan \theta}{1 - \tan^2 \theta} + \tan \theta}{1 - \left(\frac{2 \tan \theta}{1 - \tan^2 \theta}\right) \tan \theta}$

$= \dfrac{\frac{2 \tan \theta + \tan \theta - \tan^3 \theta}{1 - \tan^2 \theta}}{\frac{1 - \tan^2 \theta - 2 \tan^2 \theta}{1 - \tan^2 \theta}}$

$= \dfrac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$

13. **a)** $-\dfrac{4\sqrt{2}}{9}$ **c)** $\dfrac{\sqrt{3}}{3}$

b) $-\dfrac{7}{9}$ **d)** $-\dfrac{10\sqrt{2}}{27}$

14.

Write $\sin a$ in terms of $\frac{y}{r}$.

↓

Solve for x using the Pythagorean theorem, $x^2 + y^2 = r^2$.

↓

Choose the negative value of x since $a \in \left[\frac{\pi}{2}, \pi\right]$, and determine $\cos a$.

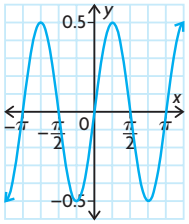
↓

Write $\cos a$ in terms of $\frac{x}{r}$.

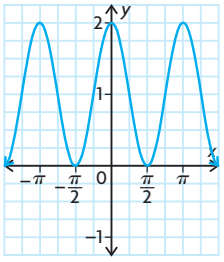
↓

Use the formula $\sin 2a = 2 \sin a \cos a$ to evaluate $\sin 2a$.

15. **a)** Use the formula $\sin 2x = 2 \sin x \cos x$ to determine that $\sin x \cos x = \dfrac{\sin 2x}{2}$.
Then graph the function $f(x) = \dfrac{\sin 2x}{2}$ by vertically compressing $f(x) = \sin x$ by a factor of $\frac{1}{2}$ and horizontally compressing it by a factor of $\frac{1}{2}$.

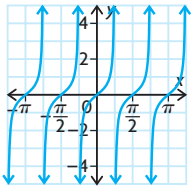


b) Use the formula $\cos 2x = 2 \cos^2 x - 1$ to determine that $2 \cos^2 x = \cos 2x + 1$.
Then graph the function $f(x) = \cos 2x + 1$ by horizontally compressing $f(x) = \cos x$ by a factor of $\frac{1}{2}$ and vertically translating it 1 unit up.



c) Use the formula $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$ to determine that $\dfrac{\tan x}{1 - \tan^2 x} = \dfrac{\tan 2x}{2}$.

Then graph the function $f(x) = \frac{\tan 2x}{2}$ by vertically compressing $f(x) = \tan x$ by a factor of $\frac{1}{2}$ and horizontally compressing it by a factor of $\frac{1}{2}$.



16. **a)** $\frac{\tan^{-1} x}{2} = \tan^{-1} y$

b) $\frac{\cos^{-1} x}{2} = \cos^{-1} y$

c) $\frac{\cos^{-1} x}{2} = \csc^{-1} y$ or $\frac{\cos^{-1} x}{2} = \sin^{-1}\left(\frac{1}{y}\right)$

d) $\frac{\sin^{-1} x}{2} = \frac{\sec^{-1} y}{4}$ or $\frac{\sin^{-1} x}{2} = \frac{\cos^{-1}\left(\frac{1}{y}\right)}{4}$

17. **a)** $x = \frac{\pi}{6}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$

b) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$, or $\frac{3\pi}{2}$

18. **a)** $\frac{2\tan\theta}{1 + \tan^2\theta}$

b) $\frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

c) $\tan\theta$

d) $\tan\theta$

Mid-Chapter Review, p. 411

1. **a)** $\cos\frac{31\pi}{16}$ **d)** $\cos\frac{7\pi}{5}$

b) $\sin\frac{2\pi}{9}$ **e)** $\sin\frac{2\pi}{7}$

c) $\tan\frac{19\pi}{10}$ **f)** $\tan\frac{7\pi}{4}$

2. $y = 6\sin x + 4$

3. **a)** $\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

b) $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

c) $\frac{1 + \tan x}{1 - \tan x}$

d) $\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x$

4. **a)** $\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

b) $\frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x}$

c) $\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$

d) $-\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x$

5. **a)** $\sqrt{3}$ **c)** $\frac{1}{2}$

b) 0 **d)** 1

6. **a)** $\tan 2x$ **d)** $\cos x$

b) $\sin x$ **e)** $\sqrt{2}(\cos x - \sin x)$

c) $\sin x$ **f)** $\frac{\tan x - 1}{1 + \tan x}$

7. $2\sqrt{3}\cos\left(x + \frac{\pi}{3}\right)$

8. **a)** $-\frac{1}{2}$ **c)** $\frac{\sqrt{2}}{2}$

b) $-\frac{1}{2}$ **d)** -1

9. **a)** $-\frac{\sqrt{11}}{11}$ **c)** $\frac{2\sqrt{10}}{11}$

b) $-\frac{\sqrt{110}}{11}$ **d)** $\frac{9}{11}$

10. $\sin 2x = \frac{24}{25}$; $\cos 2x = \frac{7}{25}$

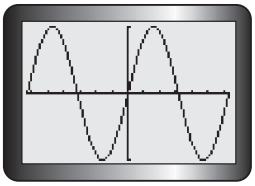
11. $\sin 2x = \frac{120}{169}$

12. $\tan 2x = \frac{24}{7}$

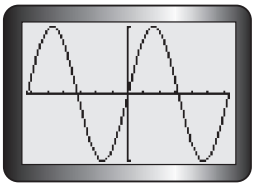
Lesson 7.4, pp. 417–418

1. Answers may vary. For example, $\sin\frac{\pi}{6} = \frac{1}{2}$; $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

2. **a)** $f(x) = \sin x$



$g(x) = \tan x \cos x$



b) $\sin x = \tan x \cos x$

c) $\tan x \cos x = \left(\frac{\sin x}{\cos x}\right)\cos x$

$= \frac{\sin x \cos x}{\cos x} = \sin x$

d) The identity is not true when $\cos x = 0$ because when $\cos x = 0$, $\tan x$, or $\frac{\sin x}{\cos x}$, is undefined.

3. **a)** C; $\sin x \cot x = \cos x$

b) D; $1 - 2\sin^2 x = 2\cos^2 x - 1$

c) B; $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

d) A; $\sec^2 x = \sin^2 x + \cos^2 x + \tan^2 x$

4. **a)** $\sin x \cot x = \cos x$

LS $= \sin x \cot x$

$= (\sin x)\left(\frac{\cos x}{\sin x}\right)$

$= \frac{\sin x \cos x}{\sin x}$

$= \cos x$

$= $ RS

b) $1 - 2\sin^2 x = 2\cos^2 x - 1$

$1 - 2\sin^2 x - 2\cos^2 x + 1 = 0$

$2 - 2\sin^2 x - 2\cos^2 x = 0$

$2 - 2(\sin^2 x + \cos^2 x) = 0$

$2 - 2(1) = 0$

$2 - 2 = 0$

$0 = 0$

c) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

LS $= (\sin x + \cos x)^2$

$= \sin^2 x + 2\sin x \cos x + \cos^2 x$

$= (\sin^2 x + \cos^2 x) + 2\sin x \cos x$

$= 1 + 2\sin x \cos x$

$= $ RS

d) $\sec^2 x = \sin^2 x + \cos^2 x + \tan^2 x$

RS $= \sin^2 x + \cos^2 x + \tan^2 x$

$= (\sin^2 x + \cos^2 x) + \tan^2 x$

$= 1 + \tan^2 x$

$= \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x}$

$= \frac{\sin^2 x + \cos^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x}$

$= \sec^2 x$

$= $ LS

5. **a)** Answers may vary. For example, $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$; $\frac{1}{\cos\frac{\pi}{6}} = \frac{2\sqrt{3}}{3}$.

b) Answers may vary. For example,

$1 - \tan^2\left(\frac{\pi}{4}\right) = 1 - (1)^2$

$= 1 - 1 = 0$;

$\sec^2\left(\frac{\pi}{4}\right) = (\sqrt{2})^2 = 2$

c) Answers may vary. For example,

$\sin\left(\frac{\pi}{2} + \pi\right) = \sin\left(\frac{3\pi}{2}\right) = -1$;

$\cos\left(\frac{\pi}{2}\right)\cos\pi + \sin\left(\frac{\pi}{2}\right)\sin\pi$

$= (0)(-1) + (1)(0)$

$= 0 + 0 = 0$

d) Answers may vary. For example,

$$\cos\left(2\left(\frac{\pi}{3}\right)\right) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

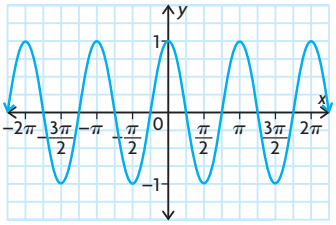
$$1 + 2\sin^2\left(\frac{\pi}{3}\right) = 1 + (2)\left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 1 + (2)\left(\frac{3}{4}\right)$$

$$= 1 + \frac{6}{4} = \frac{10}{4}$$

$$= \frac{5}{2}$$

6. Answers may vary. For example, $\cos 2x$.



7.
$$\frac{1 - \tan^2 x}{1 + \tan^2 x} = \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\sec^2 x}$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x} \times \cos^2 x$$

$$= \cos^2 x - \sin^2 x$$

$$= \cos 2x$$

8.
$$\text{LS} = \frac{1 + \tan x}{1 + \cot x} \qquad \text{RS} = \frac{1 - \tan x}{\cot x - 1}$$

$$= \frac{1 + \tan x}{1 + \frac{1}{\tan x}} \qquad = \frac{1 - \tan x}{\frac{1}{\tan x} - 1}$$

$$= \frac{1 + \tan x}{\frac{\tan x + 1}{\tan x}} \qquad = \frac{1 - \tan x}{\frac{1 - \tan x}{\tan x}}$$

$$= \tan x \qquad = \tan x$$

Since the right side and the left side are equal, $\frac{1 + \tan x}{1 + \cot x} = \frac{1 - \tan x}{\cot x - 1}$

9. **a)**
$$\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta)(\cos\theta + \sin\theta)}$$

$$= \frac{\cos\theta - \sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= 1 - \tan\theta$$

b)
$$\text{LS} = \tan^2 x - \sin^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$$

$$= \sin^2 x\left(\frac{1}{\cos^2 x} - 1\right)$$

$$= \sin^2 x(\sec^2 x - 1)$$

$$= \sin^2 x \tan^2 x$$

$$= \text{RS}$$

So $\tan^2 x - \sin^2 x = \sin^2 x \tan^2 x$.

c)
$$\tan^2 x - \cos^2 x = \frac{1}{\cos^2 x} - 1 - \cos^2 x$$

$$\tan^2 x - \cos^2 x + \cos^2 x = \frac{1}{\cos^2 x} - 1 - \cos^2 x + \cos^2 x$$

$$\tan^2 x = \frac{1}{\cos^2 x} - 1$$

$$\tan^2 x = \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$$

$$\tan^2 x = \frac{1 - \cos^2 x}{\cos^2 x}$$

$$\tan^2 x = \frac{\sin^2 x}{\cos^2 x}$$

$$\tan^2 x = \tan^2 x$$

d)
$$\frac{1}{1 + \cos\theta} + \frac{1}{1 - \cos\theta}$$

$$= \frac{1 - \cos\theta}{(1 + \cos\theta)(1 - \cos\theta)} + \frac{1 + \cos\theta}{(1 - \cos\theta)(1 + \cos\theta)}$$

$$= \frac{1 - \cos\theta}{1 - \cos^2\theta} + \frac{1 + \cos\theta}{1 - \cos^2\theta}$$

$$= \frac{1 - \cos\theta + 1 + \cos\theta}{1 - \cos^2\theta}$$

$$= \frac{2}{1 - \cos^2\theta}$$

$$= \frac{2}{\sin^2\theta}$$

10. **a)**
$$\cos x \tan^3 x = \sin x \tan^2 x$$

$$\frac{\cos x \tan^3 x}{\tan^2 x} = \frac{\sin x \tan^2 x}{\tan^2 x}$$

$$\cos x \tan x = \sin x$$

$$\cos x\left(\frac{\sin x}{\cos x}\right) = \sin x$$

$$\sin x = \sin x$$

b)
$$\sin^2\theta + \cos^4\theta = \cos^2\theta + \sin^4\theta$$

$$\sin^2\theta + \cos^4\theta - \sin^4\theta = \cos^2\theta + \sin^4\theta - \sin^4\theta$$

$$\sin^2\theta + \cos^4\theta - \sin^4\theta = \cos^2\theta$$

$$\sin^2\theta + \cos^4\theta - \sin^4\theta - \sin^2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$$

$$(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$1 = 1$$

c)
$$(\sin x + \cos x)\left(\frac{\tan^2 x + 1}{\tan x}\right)$$

$$= \frac{1}{\cos x} + \frac{1}{\sin x}$$

$$(\sin x + \cos x)\left(\frac{\sec^2 x}{\tan x}\right)$$

$$= \frac{\sin x}{\cos x \sin x} + \frac{\cos x}{\sin x \cos x}$$

$$(\sin x + \cos x)\left(\frac{1}{\cos^2 x}\right)\left(\frac{1}{\tan x}\right)$$

$$= \frac{\sin x + \cos x}{\cos x \sin x}$$

$$(\sin x + \cos x)\left(\frac{1}{\cos^2 x}\right)\left(\frac{\cos x}{\sin x}\right)$$

$$= \frac{\sin x + \cos x}{\cos x \sin x}$$

$$(\sin x + \cos x)\left(\frac{1}{\cos x \sin x}\right)$$

$$= \frac{\sin x + \cos x}{\cos x \sin x}$$

$$\frac{\sin x + \cos x}{\cos x \sin x} = \frac{\sin x + \cos x}{\cos x \sin x}$$

d)
$$\tan^2\beta + \cos^2\beta + \sin^2\beta = \frac{1}{\cos^2\beta}$$

$$\tan^2\beta + 1 = \frac{1}{\cos^2\beta}$$

$$\tan^2\beta + 1 = \sec^2\beta$$

Since $\tan^2\beta + 1 = \sec^2\beta$ is a known identity, $\tan^2\beta + \cos^2\beta + \sin^2\beta$ must equal $\frac{1}{\cos^2\beta}$.

e)
$$\sin\left(\frac{\pi}{4} + x\right) + \sin\left(\frac{\pi}{4} - x\right) = \sqrt{2}\cos x;$$

$$\sin\frac{\pi}{4}\cos x + \cos\frac{\pi}{4}\sin x + \sin\frac{\pi}{4}\cos x - \cos\frac{\pi}{4}\sin x = \sqrt{2}\cos x;$$

$$2\sin\frac{\pi}{4}\cos x = \sqrt{2}\cos x;$$

$$(2)\left(\frac{\sqrt{2}}{2}\right)(\cos x) = \sqrt{2}\cos x;$$

$$\sqrt{2}\cos x = \sqrt{2}\cos x$$

f)
$$\sin\left(\frac{\pi}{2} - x\right)\cot\left(\frac{\pi}{2} + x\right) = -\sin x;$$

$$\sin\left(\frac{\pi}{2} - x\right)\left(\frac{\cos\left(\frac{\pi}{2} + x\right)}{\sin\left(\frac{\pi}{2} + x\right)}\right) = -\sin x;$$

$$\left(\sin\frac{\pi}{2}\cos x - \cos\frac{\pi}{2}\sin x\right) \times \left(\frac{\cos\frac{\pi}{2}\cos x - \sin\frac{\pi}{2}\sin x}{\sin\frac{\pi}{2}\cos x + \cos\frac{\pi}{2}\sin x}\right) = -\sin x;$$

$$((1)(\cos x) - (0)(\sin x)) \times \left(\frac{(0)(\cos x) - (1)(\sin x)}{(1)(\cos x) + (0)(\sin x)}\right) = -\sin x;$$

$$(\cos x - 0)\left(\frac{0 - \sin x}{\cos x + 0}\right) = -\sin x;$$

$$(\cos x)\left(-\frac{\sin x}{\cos x}\right) = -\sin x;$$

$$-\sin x = -\sin x$$

11. **a)**
$$\frac{\cos 2x + 1}{\sin 2x} = \cot x$$

$$\frac{2\cos^2 x - 1 + 1}{2\sin x\cos x} = \cot x$$

$$\frac{2\cos^2 x}{2\sin x\cos x} = \cot x$$

$$\frac{\cos x}{\sin x} = \cot x$$

$$\cot x = \cot x$$

b) $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$

$\dfrac{2\sin x\cos x}{1-(1-2\sin^2 x)} = \cot x$

$\dfrac{2\sin x\cos x}{1-1+2\sin^2 x} = \cot x$

$\dfrac{2\sin x\cos x}{2\sin^2 x} = \cot x$

$\dfrac{\cos x}{\sin x} = \cot x$

$\cot x = \cot x$

c) $(\sin x+\cos x)^2 = 1+\sin 2x;$

$\sin^2 x+\sin x\cos x+\sin x\cos x$
$+\cos^2 x = 1+2\sin x\cos x;$

$\sin^2 x+2\sin x\cos x+\cos^2 x$
$= 1+2\sin x\cos x;$

$(\cos^2 x+\sin^2 x)+2\sin x\cos x$
$= 1+2\sin x\cos x;$

$1+2\sin x\cos x = 1+2\sin x\cos x$

d) $\cos^4\theta-\sin^4\theta = \cos 2\theta$

$(\cos^2\theta+\sin^2\theta)(\cos^2\theta-\sin^2\theta)$
$= \cos^2\theta-\sin^2\theta$

$(1)(\cos^2\theta-\sin^2\theta) = \cos^2\theta-\sin^2\theta$

$\cos^2\theta-\sin^2\theta = \cos^2\theta-\sin^2\theta$

e) $\cot\theta-\tan\theta = 2\cot 2\theta$

$\dfrac{\cos\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta} = 2\dfrac{\cos 2\theta}{\sin 2\theta}$

$\dfrac{\cos^2\theta}{\sin\theta\cos\theta}-\dfrac{\sin^2\theta}{\cos\theta\sin\theta}$
$= (2)\left(\dfrac{\cos 2\theta}{2\cos\theta\sin\theta}\right)$

$\dfrac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\cos\theta\sin\theta}$

$\dfrac{\cos 2\theta}{\cos\theta\sin\theta} = \dfrac{\cos 2\theta}{\cos\theta\sin\theta}$

f) $\cot\theta+\tan\theta = 2\csc 2\theta$

$\dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta} = 2\dfrac{1}{\sin 2\theta}$

$\dfrac{\cos^2\theta}{\sin\theta\cos\theta}+\dfrac{\sin^2\theta}{\cos\theta\sin\theta}$
$= (2)\left(\dfrac{1}{2\cos\theta\sin\theta}\right)$

$\dfrac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta} = \dfrac{1}{\cos\theta\sin\theta}$

$\dfrac{1}{\cos\theta\sin\theta} = \dfrac{1}{\cos\theta\sin\theta}$

g) $\dfrac{1+\tan x}{1-\tan x} = \tan\left(x+\dfrac{\pi}{4}\right)$

$\dfrac{1+\tan x}{1-\tan x} = \dfrac{\tan x+\tan\frac{\pi}{4}}{1-\tan x\tan\frac{\pi}{4}}$

$\dfrac{1+\tan x}{1-\tan x} = \dfrac{\tan x+1}{1-(\tan x)(1)}$

$\dfrac{1+\tan x}{1-\tan x} = \dfrac{1+\tan x}{1-\tan x}$

h) $\csc 2x+\cot 2x = \cot x;$

$\dfrac{1}{\sin 2x}+\dfrac{1}{\tan 2x} = \cot x;$

$\dfrac{1}{2\sin x\cos x}+\dfrac{1}{\frac{2\tan x}{1-\tan^2 x}} = \cot x;$

$\dfrac{1}{2\sin x\cos x}+\dfrac{1-\tan^2 x}{2\tan x} = \cot x;$

$\dfrac{1}{2\sin x\cos x}+\dfrac{1-\tan^2 x}{2\frac{\sin x}{\cos x}} = \cot x;$

$\dfrac{1}{2\sin x\cos x}+\dfrac{(\cos x)(1-\tan^2 x)}{2\sin x}$
$= \dfrac{\cos x}{\sin x};$

$\dfrac{1}{2\sin x\cos x}+\dfrac{(\cos x)(1-\tan^2 x)(\cos x)}{2\sin x\cos x}$
$= \dfrac{(\cos x)(2\cos x)}{(\sin x)(2\cos x)};$

$\dfrac{1}{2\sin x\cos x}+\dfrac{(\cos^2 x)(1-\tan^2 x)}{2\sin x\cos x}$
$= \dfrac{2\cos^2 x}{2\sin x\cos x};$

$\dfrac{1}{2\sin x\cos x}+\dfrac{\cos^2 x-(\tan^2 x)(\cos^2 x)}{2\sin x\cos x}$
$= \dfrac{2\cos^2 x}{2\sin x\cos x};$

$\dfrac{1}{2\sin x\cos x}+\dfrac{\cos^2 x-\sin^2 x}{2\sin x\cos x}$
$= \dfrac{2\cos^2 x}{2\sin x\cos x};$

$\dfrac{1+\cos^2 x-\sin^2 x}{2\sin x\cos x} = \dfrac{2\cos^2 x}{2\sin x\cos x};$

$\dfrac{1+\cos^2 x-\sin^2 x}{2\sin x\cos x}-\dfrac{2\cos^2 x}{2\sin x\cos x}$
$= \dfrac{2\cos^2 x}{2\sin x\cos x}-\dfrac{2\cos^2 x}{2\sin x\cos x};$

$\dfrac{1+\cos^2 x-\sin^2 x-2\cos^2 x}{2\sin x\cos x} = 0;$

$\dfrac{1-\sin^2 x-\cos^2 x}{2\sin x\cos x} = 0;$

$\dfrac{1-(\sin^2 x+\cos^2 x)}{2\sin x\cos x} = 0;$

$\dfrac{1-1}{2\sin x\cos x} = 0;$

$\dfrac{0}{2\sin x\cos x} = 0;$

$0 = 0$

i) $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

$\dfrac{2\tan x}{\sec^2 x} = \sin 2x$

$\dfrac{2\tan x}{\frac{1}{\cos^2 x}} = \sin 2x$

$(2\tan x)(\cos^2 x) = \sin 2x$

$\left(\dfrac{2\sin x}{\cos x}\right)(\cos^2 x) = \sin 2x$

$\sin 2x = 2\sin x\cos x$

Since $\sin 2x = 2\sin x\cos x$ is a known identity, $\dfrac{2\tan x}{1-\tan^2 x}$ must equal $\sin 2x$.

j) $\sec 2t = \dfrac{\csc t}{\csc t-2\sin t}$

$\dfrac{1}{\cos 2t} = \dfrac{\frac{1}{\sin t}}{\frac{1}{\sin t}-2\sin t}$

$\dfrac{1}{\cos 2t} = \dfrac{\frac{1}{\sin t}}{\frac{1}{\sin t}-\frac{2\sin^2 t}{\sin t}}$

$\dfrac{1}{\cos 2t} = \dfrac{\frac{1}{\sin t}}{\frac{1-2\sin^2 t}{\sin t}}$

$\dfrac{1}{\cos 2t} = \dfrac{1}{\sin t}\times\dfrac{\sin t}{1-2\sin^2 t}$

$\dfrac{1}{\cos 2t} = \dfrac{1}{1-2\sin^2 t}$

$\dfrac{1}{\cos 2t} = \dfrac{1}{\cos 2t}$

k) $\csc 2\theta = \dfrac{1}{2}\sec\theta\csc\theta$

$\dfrac{1}{\sin 2\theta} = \left(\dfrac{1}{2}\right)\left(\dfrac{1}{\cos\theta}\right)\left(\dfrac{1}{\sin\theta}\right)$

$\dfrac{1}{\sin 2\theta} = \dfrac{1}{2\cos\theta\sin\theta}$

$\dfrac{1}{2\sin\theta\cos\theta} = \dfrac{1}{2\sin\theta\cos\theta}$

l) $\dfrac{1}{\cos t} = \dfrac{2\sin t\cos t}{\sin t}-\dfrac{2\cos^2 t-1}{\cos t}$

$\dfrac{\sin t}{\cos t\sin t} = \dfrac{2\sin t\cos^2 t}{\sin t\cos t}$
$-\dfrac{(\sin t)(2\cos^2 t-1)}{\cos t\sin t}$

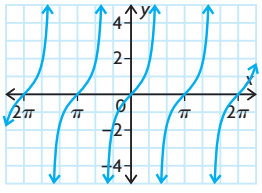
$\dfrac{\sin t}{\cos t\sin t} = \dfrac{2\sin t\cos^2 t}{\sin t\cos t}$
$-\dfrac{2\cos^2 t\sin t-\sin t}{\cos t\sin t}$

$\dfrac{\sin t}{\cos t\sin t} = \dfrac{2\sin t\cos^2 t}{\sin t\cos t}$
$-\dfrac{2\cos^2 t\sin t-\sin t}{\sin t\cos t}$

$\dfrac{\sin t}{\cos t\sin t} = \dfrac{2\sin t\cos^2 t}{\sin t\cos t}$
$+\dfrac{-2\sin t\cos^2 t+\sin t}{\sin t\cos t}$

$\dfrac{\sin t}{\cos t\sin t} = \dfrac{\sin t}{\cos t\sin t}$

12. Answers may vary. For example, an equivalent expression is $\tan x$.



13.
$$\frac{\sin x + \sin 2x}{1 + \cos x + \cos 2x} = \tan x$$
$$\frac{\sin x + 2 \sin x \cos x}{1 + \cos x + \cos 2x} = \tan x$$
$$\frac{\sin x(1 + 2 \cos x)}{1 + \cos x + \cos 2x} = \tan x$$
$$\frac{\sin x(1 + 2 \cos x)}{\cos x + (1 + \cos 2x)} = \tan x$$
$$\frac{\sin x(1 + 2 \cos x)}{\cos x + 2 \cos^2 x} = \tan x$$
$$\frac{\sin x(1 + 2 \cos x)}{\cos x(1 + 2 \cos x)} = \tan x$$
$$\frac{\sin x}{\cos x} = \tan x$$
$$\tan x = \tan x$$

14.

Definition	**Methods of Proof**
A statement of the equivalence of two trigonometric expressions	Both sides of the equation must be shown to be equivalent through graphing or simplifying/rewriting.
Trigonometric Identities	
Examples	**Non-Examples**
$\cos 2x + \sin^2 x = \cos^2 x$ $\cos 2x + 1 = 2 \cos^2 x$	$\cos 2x - 2 \sin^2 x = 1$ $\cot^2 x + \csc^2 x = 1$

15. She can determine whether the equation $2 \sin x \cos x = \cos 2x$ is an identity by trying to simplify and/or rewrite the left side of the equation so that it is equivalent to the right side of the equation. Alternatively, she can graph the functions $y = 2 \sin x \cos x$ and $y = \cos 2x$ and see if the graphs are the same. If they're the same, it's an identity, but if they're not the same, it's not an identity. By doing this she can determine it's not an identity, but she can make it an identity by changing the equation to $2 \sin x \cos x = \sin 2x$.

16. **a)** $a = 2, b = 1, c = 1$
b) $a = -1, b = 2, c = -2$

17. $\cos 4x + 4 \cos 2x + 3$; $a = 1$, $b = 4$, $c = 3$

Lesson 7.5, pp. 426–428

1. **a)** $\frac{\pi}{2}$
b) $\frac{3\pi}{2}$
c) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$
d) $\frac{7\pi}{6}$ or $\frac{11\pi}{6}$
e) 0, π, or 2π
f) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$

2. **a)** 0 or 2π
b) π
c) $\frac{\pi}{3}$ or $\frac{5\pi}{3}$
d) $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$
e) $\frac{\pi}{2}$ or $\frac{3\pi}{2}$
f) $\frac{\pi}{6}$ or $\frac{11\pi}{6}$

3. **a)** 2
b) quadrants I and II
c) $x = \frac{\pi}{3}$
d) $x = \frac{\pi}{3}$ and $\frac{2\pi}{3}$

4. **a)** 2
b) quadrants II and III
c) 30°
d) $x = 150°$ or $210°$

5. **a)** 2
b) quadrants I and III
c) 1.22
d) $\theta = 1.22$ or 4.36

6. **a)** $\theta = \frac{\pi}{4}$ or $\frac{5\pi}{4}$
b) $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$
c) $\theta = \frac{\pi}{6}$ or $\frac{11\pi}{6}$
d) $\theta = \frac{4\pi}{3}$ or $\frac{5\pi}{3}$
e) $\theta = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$
f) $\theta = \frac{\pi}{3}$ or $\frac{4\pi}{3}$

7. **a)** $\theta = 210°$ or $330°$
b) $\theta = 131.8°$ or $228.2°$
c) $\theta = 56.3°$ or $236.3°$
d) $\theta = 221.8°$ or $318.2°$
e) $\theta = 78.5°$ or $281.5°$
f) $\theta = 116.6°$ or $296.6°$

8. **a)** $x = 0.52$ or 2.62
b) $x = 0.52$ or 5.76
c) $x = 1.05$ or 5.24
d) $x = 3.67$ or 5.76

9. **a)** $x = 0.79$ or 3.93
b) $x = 0.52$ or 2.62
c) $x = 0$ or 6.28
d) $x = 3.67$ or 5.76
e) $x = 1.16$ or 5.12
f) $x = 1.11$ or 4.25

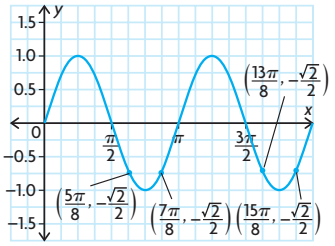
10. **a)** $x = 0.39, 1.18, 3.53$, or 4.32
b) $x = 0.13, 0.65, 1.70, 2.23, 3.27, 3.80, 4.84$, or 5.37
c) $x = 1.40, 1.75, 3.49, 3.84, 5.59$, or 5.93
d) $x = 0.59, 0.985, 2.16, 2.55, 3.73, 4.12, 5.304$, or 5.697
e) $x = 1.05, 2.09, 4.19$, or 5.24
f) $x = 1.05$

11. from about day 144 to about day 221

12. $1.86 \text{ s} < t < 4.14 \text{ s}$;
$9.86 \text{ s} < t < 12.14 \text{ s}$;
$17.86 \text{ s} < t < 20.14 \text{ s}$

13. $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

14.



15. The value of $f(x) = \sin x$ is the same at x and $\pi - x$. In other words, it is the same at x and half the period minus x. Since the period of $f(x) = 25 \sin\frac{\pi}{50}(x + 20) - 55$ is 100, if the function were not horizontally translated, its value at x would be the same as at $50 - x$. The function is horizontally translated 20 units to the left, however, so it goes through half its period from $x = -20$ to $x = 30$. At $x = 3$, the function is 23 units away from the left end of the range, so it will have the same value at $x = 30 - 23$ or $x = 7$, which is 23 units away from the right end of the range.

16. To solve a trigonometric equation **algebraically**, first isolate the trigonometric function on one side of the equation. For example, the trigonometric equation $5 \cos x - 3 = 2$ would become $5 \cos x = 5$, which would then become $\cos x = 1$. Next, apply the inverse of the trigonometric function to both sides of the equation. For example, the trigonometric equation $\cos x = 1$ would become $x = \cos^{-1} 1$. Finally, simplify the equation. For example, $x = \cos^{-1} 1$ would become $x = 0 + 2n\pi$, where $n \in \mathbf{I}$.
To solve a trigonometric equation **graphically**, first isolate the trigonometric function on one side of the equation. For example, the trigonometric equation $5 \cos x - 3 = 2$ would become $5 \cos x = 5$, which would then become $\cos x = 1$. Next, graph both sides of the equation. For example, the functions $f(x) = \cos x$ and $f(x) = 1$ would both be graphed. Finally, find the points where the two graphs intersect. For example, $f(x) = \cos x$ and $f(x) = 1$ would intersect at $x = 0 + 2n\pi$, where $n \in \mathbf{I}$.
Similarity: Both trigonometric functions are first isolated on one side of the equation.
Differences: The inverse of a trigonometric function is not applied in the graphical method, and the points of intersection are not obtained in the algebraic method.

17. $x = 0 + n\pi, \frac{2\pi}{3} + 2n\pi$, and $\frac{4\pi}{3} + n\pi$, where $n \in \mathbf{I}$

18. **a)** $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{5\pi}{4}$, or $\frac{3\pi}{4}$

b) $x = \frac{\pi}{6}, \frac{\pi}{2}$, or $\frac{5\pi}{6}$

Lesson 7.6, pp. 435–437

1. **a)** $(\sin\theta)(\sin\theta - 1)$
b) $(\cos\theta - 1)(\cos\theta - 1)$
c) $(3\sin\theta + 2)(\sin\theta - 1)$
d) $(2\cos\theta - 1)(2\cos\theta + 1)$
e) $(6\sin x - 2)(4\sin x + 1)$
f) $(7\tan x + 8)(7\tan x - 8)$

2. **a)** $y = \pm\frac{\sqrt{3}}{3}, x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

b) $y = 0$ or $-1, x = 0, \pi, \frac{3\pi}{2}$, or 2π

c) $y = 0$ or $z = \frac{1}{2}, x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$

d) $y = 0$ or $z = 1, x = 0, \pi$, or 2π

3. **a)** $y = \frac{1}{3}$ or $\frac{1}{2}$

b) $x = 1.05, 1.91, 4.37$, or 5.24

4. **a)** $\theta = 90°$ or $270°$
b) $\theta = 0°, 180°$, or $360°$
c) $\theta = 45°, 135°, 225°$, or $315°$
d) $\theta = 60°, 120°, 240°$, or $300°$
e) $\theta = 30°, 150°, 210°$, or $330°$
f) $\theta = 45°, 135°, 225°$, or $315°$

5. **a)** $x = 0°, 90°, 180°, 270°$, or $360°$
b) $x = 0°, 180°$, or $360°$
c) $x = 90°$ or $270°$
d) $x = 60°, 90°, 120°$, or $270°$
e) $x = 45°, 135°, 225°$, or $315°$
f) $x = 90°$ or $180°$

6. **a)** $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$

b) $x = \frac{3\pi}{2}$

c) $x = 0, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}$, or 2π

d) $x = \frac{\pi}{3}, \frac{4\pi}{3}$, or $\frac{5\pi}{3}$

e) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$, or $\frac{7\pi}{4}$

f) $x = 0, \frac{3\pi}{2}$, or 2π

7. **a)** $\theta = \frac{\pi}{3}, \pi$, or $\frac{5\pi}{3}$

b) $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$, or $\frac{3\pi}{2}$

c) $\theta = \pi$

d) $\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$

e) $\theta = \frac{\pi}{4}, 2.82, \frac{5\pi}{4}$, or 5.96

f) $\theta = 0.73, 2.41, 3.99$, or 5.44

8. **a)** $x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$

b) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

c) $x = 0, 0.96\,\pi, 5.33$, or 2π

d) $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$

e) $x = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{3\pi}{2}$, or $\frac{7\pi}{4}$

f) $x = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$, or 2π

9. **a)** $x = \frac{\pi}{3}, 1.98, 4.30$, or $\frac{5\pi}{3}$

b) $x = 0.45, \frac{2\pi}{3}, \frac{4\pi}{3}$, or 5.83

c) $x = \frac{\pi}{6}, 0.85, \frac{5\pi}{6}$, or 2.29

d) $x = \frac{\pi}{2}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$

10. $x = 0.15, 1.02, 2.12$, or 2.99

11. $b = 1 + \sqrt{3}, c = \sqrt{3}$

12. $c = \frac{1}{2}$

13. $\frac{\pi}{3}\text{km} < d < \frac{2\pi}{3}\text{km}$, $\frac{4\pi}{3}\text{km} < d < \frac{5\pi}{3}\text{km}$

14. $x = 1.91$ or 4.37

15. **a)** $x = \frac{3\pi}{4}$ or $\frac{5\pi}{4}$

b) $x = \frac{3\pi}{4} + 2n\pi$ or $\frac{5\pi}{4} + 2n\pi$, where $n \in \mathbf{I}$

16. It is possible to have different numbers of solutions for quadratic trigonometric equations because, when factored, a quadratic trigonometric equation can be one expression multiplied by another expression or it can be a single expression squared. For example, the equation $\cos^2 x + \frac{3}{2}\cos x + \frac{1}{2}$ becomes $(\cos x + 1)\left(\cos x + \frac{1}{2}\right)$ when factored, and it has the solutions $\frac{2\pi}{3}$, π, and $\frac{4\pi}{3}$ in the interval $0 \le x \le 2\pi$. In comparison, the equation $\cos^2 x + 2\cos x + 1 = 0$ becomes $(\cos x + 1)^2$ when factored, and it has only one solution, π, in the interval $0 \le x \le 2\pi$. Also, different expressions produce different numbers of solutions. For example, the expression $\cos x + \frac{1}{2}$ produces two solutions in the interval $0 \le x \le 2\pi$ $\left(\frac{2\pi}{3}\text{ and }\frac{4\pi}{3}\right)$ because $\cos x = -\frac{1}{2}$ for two different values of x. The expression $\cos x + 1$, however, produces only one solution in the interval $0 \le x \le 2\pi$ (π), because $\cos x = -1$ for only one value of x.

17. $a = \frac{\pi}{4}, \frac{5\pi}{4}$

18. $x = 0.72, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$, or 5.56

19. $x = 15°, 75°, 105°, 165°, 195°, 255°, 285°$, or $345°$

20. $\theta = 0.96$

Chapter Review, p. 440

1. **a)** Answers may vary. For example, $\sin\frac{7\pi}{10}$.
b) Answers may vary. For example, $\cos\frac{8\pi}{7}$.
c) Answers may vary. For example, $\sin\frac{6\pi}{7}$.
d) Answers may vary. For example, $\cos\frac{\pi}{7}$.

2. $y = 5\cos(x) - 8$

3. **a)** $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$

b) $-\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$

c) $\frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x}$

d) $-\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x$

4. **a)** $-\frac{\sqrt{3}}{3}$ **b)** $-\frac{\sqrt{3}}{2}$

5. **a)** $\frac{1}{2}$ **c)** $-\frac{\sqrt{2}}{2}$
b) $\frac{\sqrt{3}}{2}$ **d)** $\sqrt{3}$

6. **a)** $\sin 2x = \frac{24}{25}, \cos 2x = \frac{7}{25}$, $\tan 2x = \frac{24}{7}$

b) $\sin 2x = -\frac{336}{625}, \cos 2x = -\frac{527}{625}$, $\tan 2x = \frac{336}{527}$

c) $\sin 2x = -\frac{120}{169}, \cos 2x = \frac{119}{169}$, $\tan 2x = -\frac{120}{119}$

7. **a)** trigonometric identity
b) trigonometric equation
c) trigonometric identity
d) trigonometric equation

8. $\frac{\cos^2 x}{\cot^2 x} = 1 - \cos^2 x$
$\frac{\frac{\cos^2 x}{\cos^2 x}}{\sin^2 x} = 1 - \cos^2 x$
$\frac{(\cos^2 x)(\sin^2 x)}{\cos^2 x} = 1 - \cos^2 x$
$\sin^2 x = 1 - \cos^2 x$
$1 - \cos^2 x = 1 - \cos^2 x$

9. $\dfrac{2(\sec^2 x - \tan^2 x)}{\csc x} = \sin 2x \sec x$

$\dfrac{2(1)}{\csc x} = \sin 2x \sec x$

$\dfrac{2}{\csc x} = \sin 2x \sec x$

$2 \sin x = \sin 2x \sec x$

$\dfrac{2 \sin x \cos x}{\cos x} = \sin 2x \sec x$

$\dfrac{\sin 2x}{\cos x} = \sin 2x \sec x$

$\sin 2x \sec x = \sin 2x \sec x$

10. **a)** $x = \dfrac{7\pi}{6}$ or $\dfrac{11\pi}{6}$

b) $x = \dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$

c) $x = \dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$

11. **a)** $y = -2$ or 2

b) $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$, or $\dfrac{11\pi}{6}$

12. **a)** $x = \dfrac{\pi}{2}, \dfrac{7\pi}{6}$, or $\dfrac{11\pi}{6}$

b) $x = 0, \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \pi, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$, or 2π

c) $x = \dfrac{\pi}{4}, \dfrac{2\pi}{3}, \dfrac{4\pi}{3}$, or $\dfrac{7\pi}{4}$

d) $x = 0.95$ or 4.09

13. $x = \dfrac{\pi}{2}, \pi$, or $\dfrac{3\pi}{2}$

Chapter Self-Test, p. 441

1. $\dfrac{1 - 2 \sin^2 x}{\cos x + \sin x} + \sin x = \cos x$

$\dfrac{1 - 2 \sin^2 x}{\cos x + \sin x} + \sin x - \sin x$

$= \cos x - \sin x$

$\dfrac{1 - 2 \sin^2 x}{\cos x + \sin x} = \cos x - \sin x$

$1 - 2 \sin^2 x = (\cos x - \sin x) \times (\cos x + \sin x)$

$\cos 2x = (\cos x - \sin x) \times (\cos x + \sin x)$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \cos 2x$

2. all real numbers x, where $0 \leq x \leq 2\pi$

3. **a)** $x = \dfrac{\pi}{6}$ or $x = \dfrac{11\pi}{6}$

b) $x = \dfrac{2\pi}{3}$ or $x = \dfrac{5\pi}{3}$

c) $x = \dfrac{5\pi}{4}$ or $x = \dfrac{7\pi}{4}$

4. $a = 2$, $b = 1$

5. $t = 7$, 11, 19, and 23

6. Nina can find the cosine of $\frac{11\pi}{4}$ by using the formula $\cos(x + y) = \cos x \cos y - \sin x \sin y$. The cosine of π is -1, and the cosine of $\frac{7\pi}{4}$ is $\frac{\sqrt{2}}{2}$. Also, the sine of π is 0, and the sine of $\frac{7\pi}{4}$ is $-\frac{\sqrt{2}}{2}$. Therefore,

$\cos \dfrac{11\pi}{4} = \cos\left(\pi + \dfrac{7\pi}{4}\right)$

$= \left(-1 \times \dfrac{\sqrt{2}}{2}\right) - \left(0 \times -\dfrac{\sqrt{2}}{2}\right)$

$= -\dfrac{\sqrt{2}}{2} - 0$

$= -\dfrac{\sqrt{2}}{2}$

7. $x = 3.31$ or 6.12

8. $-\dfrac{33}{65}, -\dfrac{16}{65}$

9. **a)** $-\dfrac{4\sqrt{5}}{9}$ **b)** $\dfrac{1}{9}$ **c)** $\sqrt{\dfrac{3 - \sqrt{5}}{6}}$ **d)** $\dfrac{22}{27}$

10. **a)** $x = -\dfrac{5\pi}{3}, -\dfrac{\pi}{3}, \dfrac{\pi}{3}$, or $\dfrac{5\pi}{3}$

b) $x = -\dfrac{4\pi}{3}, -\dfrac{2\pi}{3}, \dfrac{2\pi}{3}$, or $\dfrac{4\pi}{3}$

c) $x = -\pi$ and π

Chapter 8

Getting Started, p. 446

1. **a)** $\dfrac{1}{5^2} = \dfrac{1}{25}$

b) 1

c) $\sqrt{36} = 6$

d) $\sqrt[3]{125} = 5$

e) $-\sqrt{121} = -11$

f) $\left(\sqrt[3]{\dfrac{27}{8}}\right)^2 = \dfrac{9}{4}$

2. **a)** $3^7 = 2187$

b) $(-2)^2 = 4$

c) $10^3 = 1000$

d) $7^4 = 2401$

e) $8^{\frac{2}{3}} = 4$

f) $4^{\frac{1}{2}} = \sqrt{4} = 2$

3. **a)** $8m^3$

b) $\dfrac{1}{a^8 b^{10}}$

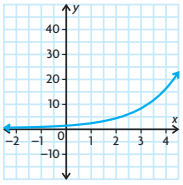
c) $4|x|^3$

d) x^3y

e) $-d^2c^2$

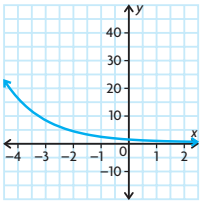
f) x

4. **a)**



D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} | y > 0\}$, y-intercept 1, horizontal asymptote $y = 0$

b)



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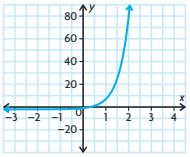
c)



D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} | y > -2\}$, y-intercept -1, horizontal asymptote $y = -2$

5. **a)** **i)** $y = \dfrac{x + 6}{3}$

ii) $y = \pm\sqrt{x + 5}$

iii) $y = \sqrt[3]{\dfrac{x}{6}}$

iv)

b) The inverses of (i) and (iii) are functions.

6. **a)** 800 bacteria

b) 6400 bacteria

c) 209 715 200

d) 4.4×10^{15}

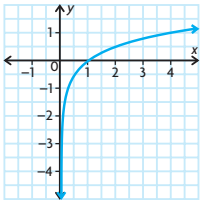
7. 12 515 people

8.

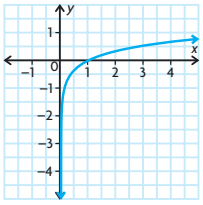
Similarities	Differences
• same y-intercept • same shape • same horizontal asymptote • both are always positive	• one is always increasing, the other is always decreasing • different end behaviour • reflections of each other across the y-axis

Lesson 8.1, p. 451

1. **a)** $x = 4^y$ or $f^{-1}(x) = \log_4 x$



b) $x = 8^y$ or $f^{-1}(x) = \log_8 x$



Answers

9. $\dfrac{2(\sec^2 x - \tan^2 x)}{\csc x} = \sin 2x \sec x$

$\dfrac{2(1)}{\csc x} = \sin 2x \sec x$

$\dfrac{2}{\csc x} = \sin 2x \sec x$

$2 \sin x = \sin 2x \sec x$

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11. **a)** $y = -2$ or 2

b) $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}$, or $\dfrac{11\pi}{6}$

12. **a)** $x = \dfrac{\pi}{2}, \dfrac{7\pi}{6}$, or $\dfrac{11\pi}{6}$

b) $x = 0, \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \pi, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$, or 2π

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Chapter Self-Test, p. 441

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Chapter 8

Getting Started, p. 446

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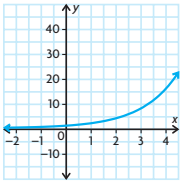
4. **a)**



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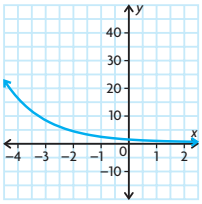
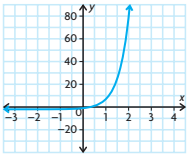
b)



D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} | y > 0\}$, y-intercept 1, horizontal asymptote $y = 0$

c)



D $= \{x \in \mathbf{R}\}$, R $= \{y \in \mathbf{R} | y > -2\}$, y-intercept -1, horizontal asymptote $y = -2$

5. **a)** **i)** $y = \dfrac{x + 6}{3}$

ii) $y = \pm\sqrt{x + 5}$

iii) $y = \sqrt[3]{\dfrac{x}{6}}$

iv)

b) The inverses of (i) and (iii) are functions.

6. **a)** 800 bacteria

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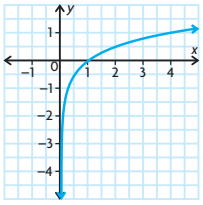
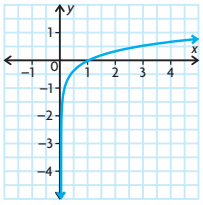
Similarities	Differences
• same y-intercept • same shape • same horizontal asymptote • both are always positive	• one is always increasing, the other is always decreasing • different end behaviour • reflections of each other across the y-axis

Lesson 8.1, p. 451

1. **a)** $x = 4^y$ or $f^{-1}(x) = \log_4 x$

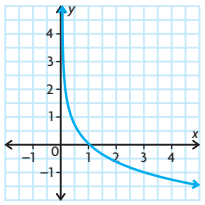


b) $x = 8^y$ or $f^{-1}(x) = \log_8 x$

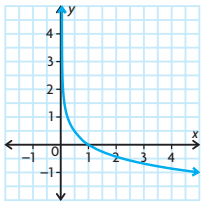


Answers

c) $x = \left(\frac{1}{3}\right)^y$ or $f^{-1}(x) = \log_{\left(\frac{1}{3}\right)} x$



d) $x = \left(\frac{1}{5}\right)^y$ or $f^{-1}(x) = \log_{\left(\frac{1}{5}\right)} x$



2. **a)** **i)** $x = 4^y$
ii) $\log_4 x = y$
b) **i)** $x = 8^y$
ii) $\log_8 x = y$
c) **i)** $x = \left(\frac{1}{3}\right)^y$
ii) $\log_{\frac{1}{3}} x = y$
d) **i)** $x = \left(\frac{1}{5}\right)^y$
ii) $\log_{\frac{1}{5}} x = y$

3. All the graphs have the same basic shape, but the last two are reflected over the x-axis, compared with the first two. All the graphs have the same x-intercept, 1. All have the same vertical asymptote, $x = 0$.

4. Locate the point on the graph that has 8 as its x-coordinate. This point is (8, 3). The y-coordinate of this point is the solution to $2^y = 8$, $y = 3$.

5. **a)** $x = 3^y$ **c)** $x = \left(\frac{1}{4}\right)^y$
b) $x = 10^y$ **d)** $x = m^y$

6. **a)** $\log_3 x = y$ **c)** $\log_{\frac{1}{4}} x = y$
b) $\log_{10} x = y$ **d)** $\log_m x = y$

7. **a)** $x = 5^y$ **c)** $x = 3^y$
b) $x = 10^y$ **d)** $x = \frac{1^y}{4}$

8. **a)** $y = 5^x$ **c)** $y = 3^x$
b) $y = 10^x$ **d)** $y = \frac{1^x}{4}$

9. **a)** 2 **d)** 0
b) 3 **e)** -1
c) 4 **f)** $\frac{1}{2}$

10. Since 3 is positive, no exponent for 3^x can produce -9.

11. **a)** $\left(\frac{1}{4}, -2\right), \left(\frac{1}{2}, -1\right), (1, 0), (2, 1), (4, 2)$
b) $\left(\frac{1}{100}, -2\right), \left(\frac{1}{10}, -1\right), (1, 0), (10, 1), (100, 2)$

Lesson 8.2, pp. 457–458

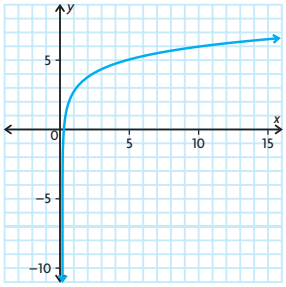
1. **a)** vertical stretch by a factor of 3
b) horizontal compression by a factor of $\frac{1}{2}$
c) vertical translation 5 units down
d) horizontal translation 4 units left

2. **a)** **(a)** $\left(\frac{1}{10}, -3\right), (1, 0), (10, 3)$
(b) $\left(\frac{1}{20}, -1\right), \left(\frac{1}{2}, 0\right), (5, 1)$
(c) $\left(\frac{1}{10}, -6\right), (1, -5), (10, -4)$
(d) $\left(-3\frac{9}{10}, -1\right), (-3, 0), (6, 1)$
b) **(a)** D $= \{x \in \mathbf{R} \mid x > 0\}$, R $= \{y \in \mathbf{R}\}$
(b) D $= \{x \in \mathbf{R} \mid x > 0\}$, R $= \{y \in \mathbf{R}\}$
(c) D $= \{x \in \mathbf{R} \mid x > 0\}$, R $= \{y \in \mathbf{R}\}$
(d) D $= \{x \in \mathbf{R} \mid x > -4\}$, R $= \{y \in \mathbf{R}\}$

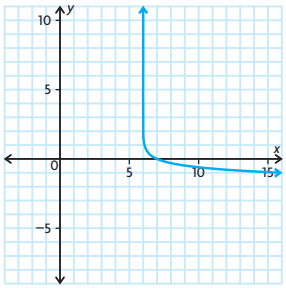
3. **a)** $f(x) = 5 \log_{10} x + 3$
b) $f(x) = -\log_{10}(3x)$
c) $f(x) = \log_{10}(x + 4) - 3$
d) $f(x) = -\log_{10}(x - 4)$

4. **i)** **a)** reflection in the x-axis and a vertical stretch by a factor of 4; $c = 5$ resulting in a translation 5 units up
b) (1, 5), (10, 1)
c) vertical asymptote is $x = 0$
d) D $= \{x \in \mathbf{R} \mid x > 0\}$, R $= \{y \in \mathbf{R}\}$
ii) **a)** vertical compression by a factor of $\frac{1}{2}$; $d = 6$ resulting in a horizontal translation 6 units to the right; $c = 3$ resulting in a vertical translation 3 units up
b) $(7, 3), \left(16, 3\frac{1}{2}\right)$
c) vertical asymptote is $x = 6$
d) D $= \{x \in \mathbf{R} \mid x > 6\}$, R $= \{y \in \mathbf{R}\}$
iii) **a)** horizontal compression by a factor of $\frac{1}{3}$; $c = -4$ resulting in a vertical shift 4 units down
b) $\left(\frac{1}{3}, -4\right), \left(3\frac{1}{3}, -3\right)$
c) vertical asymptote is $x = 0$
d) D $= \{x \in \mathbf{R} \mid x > 6\}$, R $= \{y \in \mathbf{R}\}$
iv) **a)** vertical stretch by a factor of 2; $k = -2$ resulting in a horizontal compression by a factor of $\frac{1}{2}$ and a reflection in the y-axis; $d = -2$ resulting in a horizontal translation 2 units to the left.
b) $\left(-2\frac{1}{2}, 0\right), (-7, 2)$
c) vertical asymptote is $x = -2$
d) D $= \{x \in \mathbf{R} \mid x < -2\}$, R $= \{y \in \mathbf{R}\}$
v) **a)** horizontal compression by a factor of $\frac{1}{2}$; $d = -2$ resulting in a horizontal translation 2 units to the left
b) $\left(-1\frac{1}{2}, 0\right), (3, 1)$
c) vertical asymptote is $x = -2$
d) D $= \{x \in \mathbf{R} \mid x > -2\}$, R $= \{y \in \mathbf{R}\}$
vi) **a)** reflection in the x-axis; $d = -2$, resulting in a horizontal translation 2 units to the right
b) $(-3, 0), (-12, 1)$
c) vertical asymptote is $x = -2$
d) D $= \{x \in \mathbf{R} \mid x < -2\}$, R $= \{y \in \mathbf{R}\}$

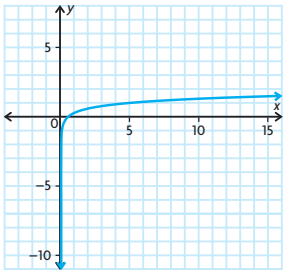
5. **a)** D $= \{x \in \mathbf{R} \mid x > 0\}$, R $= \{y \in \mathbf{R}\}$



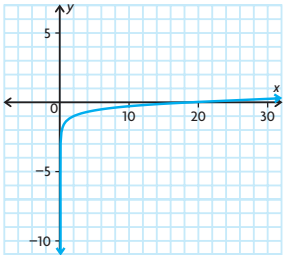
b) D $= \{x \in \mathbf{R} \mid x > -6\}$, R $= \{y \in \mathbf{R}\}$



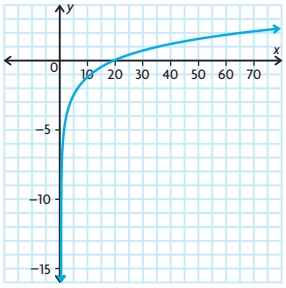
c) D $= \{x \in \mathbf{R} \mid x > 0\}$, R $= \{y \in \mathbf{R}\}$



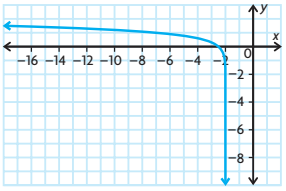
d) $D = \{x \in \mathbf{R} | x > 0\}$,
$R = \{y \in \mathbf{R}\}$



e) $D = \{x \in \mathbf{R} | x > 0\}$,
$R = \{y \in \mathbf{R}\}$



f) $D = \{x \in \mathbf{R} | x < -2\}$,
$R = \{y \in \mathbf{R}\}$



6. The functions are inverses of each other.

7. **a)** The graph of $g(x) = \log_3(x + 4)$ is the same as the graph of $f(x) = \log_3 x$, but horizontally translated 4 units to the left. The graph of $h(x) = \log_3 x + 4$ is the same as the graph of $f(x) = \log_3 x$, but vertically translated 4 units up.

b) The graph of $m(x) = 4 \log_3 x$ is the same as the graph of $f(x) = \log_3 x$, but vertically stretched by a factor of 4. The graph of $n(x) = \log_3 4x$ is the same as the graph of $f(x) = \log_3 x$, but horizontally compressed by a factor of $\frac{1}{4}$.

8. **a)** $f(x) = -3 \log_{10}\left(\frac{1}{2}x - 5\right) + 2$

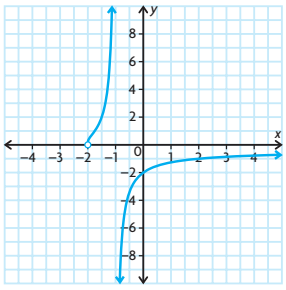
b) $(30, -1)$

c) $D = \{x \in \mathbf{R} | x > 5\}$,
$R = \{y \in \mathbf{R}\}$

9. vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left

10. domain, range, and vertical asymptote

11.



Lesson 8.3, pp. 466–468

1. **a)** $\log_4 16 = 2$
b) $\log_3 81 = 4$
c) $\log_8 1 = 0$
d) $\log_6 \frac{1}{36} = -2$
e) $\log_{\frac{1}{3}} \frac{1}{27} = 3$
f) $\log_8 2 = \frac{1}{3}$

2. **a)** $2^3 = 8$
b) $5^{-2} = \frac{1}{25}$
c) $3^4 = 81$
d) $\left(\frac{1}{6}\right)^{-3} = 216$
e) $6^{\frac{1}{2}} = \sqrt{6}$
f) $10^0 = 1$

3. **a)** 1
b) 0
c) -2
d) $\frac{1}{2}$
e) 3
f) $\frac{1}{3}$

4. **a)** -1
b) 0
c) 6
d) about 25
e) 1.78
f) 0.01

5. **a)** $\frac{1}{2}$
b) 1
c) 7
d) -2
e) $\frac{1}{3}$
f) $\frac{3}{2}$

6. **a)** 125
b) 3
c) -3
d) 16
e) $\sqrt{5}$
f) 8

7. **a)** about 2.58
b) about 3.26
c) about 4.29
d) about 4.52

8. **a)** about 2.50
b) about 2.65
c) about 4.88
d) about 2.83

9. **a)** 5
b) 25
c) $\frac{1}{16}$
d) n
e) b
f) 0

10. $\frac{4}{3}$

11. about 1.7 weeks or 12 days

12. **a)** 4.68 g **b)** 522 years

13. A:$(0.0625) = 0.017$; B:$(1) = 0.159$; B has a steeper slope.

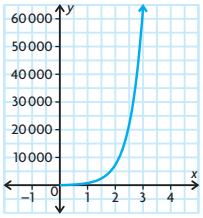
14. **a)** about 233 mph **b)** 98 miles

15. $\log 365 = 2.562$
$\frac{3}{2} \log 150 - 0.7 = 2.564$

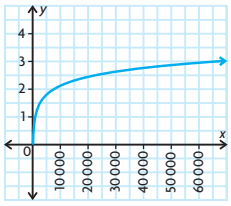
16. **a)** about 83 years
b) about 164 years

17. **a)** $y = 100(2)^{\frac{x}{0.32}}$

b)



c)



d) $y = 0.32 \log_2\left(\frac{x}{100}\right)$; this equation tells how many hours, y, it will take for the number of bacteria to reach x.

e) about 0.69 h; evaluate the inverse function for $x = 450$

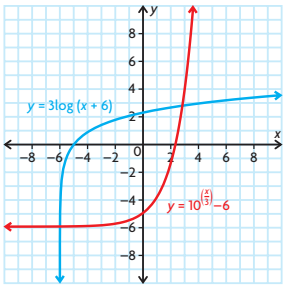
18. **a)** 1.0000
b) 3.3219
c) 2.3652
d) 2.1745
e) -0.5000
f) 2.9723

19. **a)** positive for all values $x > 1$
b) negative for all values $0 < x < 1$
c) undefined for all values $x \leq 0$

20. **a)** 1027
b) -27.14

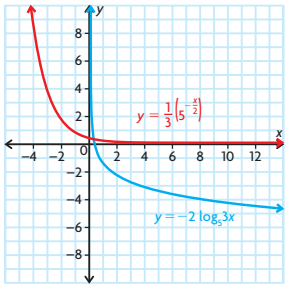
21. **a)** $y = x^3$
b) $\frac{\sqrt[3]{2}}{3}$
c) $\sqrt[(x-2)]{0.5}$
d) $2^{\frac{x-3}{3}} + 3$

22. **a)**



function: $y = 3 \log (x + 6)$ $D = \{x \in \mathbf{R} \| x > -6\}$ $R = \{y \in \mathbf{R}\}$ asymptote: $x = -6$
inverse: $y = 10^{\frac{x}{3}} - 6$ $D = \{x \in \mathbf{R}\}$ $R = \{y \in \mathbf{R} \| y > -6\}$ asymptote: $y = -6$

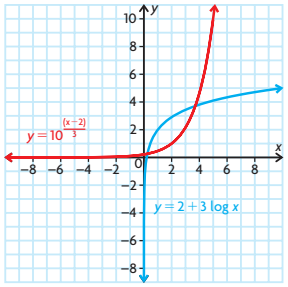
b)



function: $y = -2\log_5 3x$
D $= \{x \in \mathbf{R} | x > 0\}$
R $= \{y \in \mathbf{R}\}$
asymptote: $x = 0$

inverse: $y = \frac{1}{3}\left(5^{-\frac{x}{2}}\right)$
D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R} | y > 0\}$
asymptote: $y = 0$

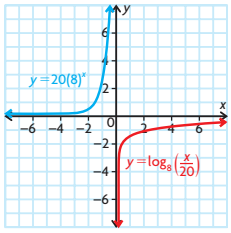
c)



function: $y = 2 + 3\log x$
D $= \{x \in \mathbf{R} | x > 0\}$
R $= \{y \in \mathbf{R}\}$
asymptote: $x = 0$

inverse: $y = 10^{\frac{(x-2)}{3}}$
D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R} | y > 0\}$
asymptote: $y = 0$

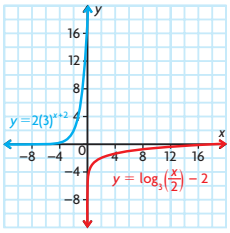
d)



function: $y = 20(8)^x$
D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R} | y > 0\}$
asymptote: $y = 0$

inverse: $y = \log_8\left(\frac{x}{20}\right)$
D $= \{x \in \mathbf{R} | x > 0\}$
R $= \{y \in \mathbf{R}\}$
asymptote: $x = 0$

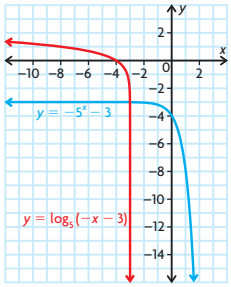
e)



function: $y = 2(3)^{x+2}$
D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R} | y > 0\}$
asymptote: $y = 0$

inverse: $y = \log_3\left(\frac{x}{2}\right) - 2$
D $= \{x \in \mathbf{R} | x > 0\}$
R $= \{y \in \mathbf{R}\}$
asymptote: $x = 0$

f)



function: $y = -5^x - 3$
D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R} | y < -3\}$
asymptote: $y = -3$

inverse: $y = \log_5(-x - 3)$
D $= \{x \in \mathbf{R} | x < -3\}$
R $= \{y \in \mathbf{R}\}$
asymptote: $x = -3$

23. Given the constraints, two integer values are possible for y, either 1 or 2. If $y = 3$, then x must be 1000, which is not permitted.

Lesson 8.4, pp. 475–476

1. **a)** $\log 45 + \log 68$
b) $\log_m p + \log_m q$
c) $\log 123 - \log 31$
d) $\log_m p - \log_m q$
e) $\log_2 14 + \log_2 9$
f) $\log_4 81 - \log_4 30$

2. **a)** $\log 35$
b) $\log_3 2$
c) $\log_m ab$
d) $\log \frac{x}{y}$
e) $\log_6 504$
f) $\log_4 6$

3. **a)** $2 \log 5$
b) $-1 \log 7$
c) $q \log_m p$
d) $\frac{1}{3} \log 45$
e) $\frac{1}{2} \log_7 36$
f) $\frac{1}{5} \log_5 125$

4. **a)** $\log_3 27$; 3
b) $\log_5 25$; 2
c) $\log 100$; 2
d) $7 \log_4 4$; 7
e) $\log_2 32$; 5
f) $\frac{1}{2} \log 10$; $\frac{1}{2}$

5. $y = \log_2(4x) = \log_2 x + \log_2 4$
$= \log_2 x + 2$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ up 2 units;
$y = \log_2(8x) = \log_2 x + \log_2 8$
$= \log_2 x + 3$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ up 3 units;
$y = \log_2\left(\frac{x}{2}\right) = \log_2 x - \log_2 2$
$= \log_2 x - 1$, so $y = \log_2(4x)$ vertically shifts $y = \log_2 x$ down 1 unit

6. **a)** 1.5
b) 2
c) 1.5
d) -0.5
e) 4
f) 2

7. **a)** $\log_b x + \log_b y + \log_b z$
b) $\log_b z - (\log_b x + \log_b y)$
c) $2 \log_b x + 3 \log_b y$
d) $\frac{1}{2}(5 \log_b x + \log_b y + 3 \log_b z)$

8. $\log_5 3$ means $5^x = 3$ and $\log_5 \frac{1}{3}$ means $5^y = \frac{1}{3}$; since $\frac{1}{3} = 3^{-1}$, $5^y = 5^{x(-1)}$;
therefore $\log_5 3 + \log_5 \frac{1}{3} = x + x(-1) = 0$

9. **a)** $\log_5 56$
b) $\log_3 2$
c) $\log_2 45$
d) $\log_3 4$
e) $\log_4(3\sqrt{2})$
f) $\log 16$

10. **a)** $\log_2 x = \log_2 245$; $x = 245$
b) $\log x = \log 432$; $x = 432$
c) $\log_4 x = \log_7 5$; $x = 5$
d) $\log_7 x = \log_7 5$; $x = 5$
e) $\log_3 x = \log_3 4$; $x = 4$
f) $\log_5 x = \log_5 384$; $x = 384$

11. **a)** $\log_2 xyz$
b) $\log_5 \frac{uw}{v}$
c) $\log_6 \frac{a}{bc}$
d) $\log_2 xy$
e) $\log_3 3x^2$
f) $\log_4 \frac{x^5}{v}$

12. $\log_a \frac{\sqrt{x}\sqrt{y}}{\sqrt[4]{z^3}}$

13. vertical stretch by a factor of 3, and vertical shift 3 units up

14. Answers may vary. For example,
$f(x) = 2 \log x - \log 12$
$g(x) = \log \frac{x^2}{12}$
$2 \log x - \log 12 = \log x^2 - \log 12$
$= \log \frac{x^2}{12}$

15. Answers may vary. For example, any number can be written as a power with a given base. The base of the logarithm is 3. Write each term in the quotient as a power of 3. The laws of logarithms make it possible to evaluate the expression by simplifying the quotient and noting the exponent.

16. $\log_x x^{m-1} + 1 = m - 1 + 1 = m$

17. $\log_b x\sqrt{x} = \log_b x + \log_b \sqrt{x}$
$= \log_b x + \frac{1}{2}\log_b x$
$= 0.3 + 0.3\left(\frac{1}{2}\right)$
$= 0.45$

18. The two functions have different domains. The first function has a domain of $x > 0$. The second function has a domain of all real numbers except 0, since x is squared.

19. Answers may vary; for example,
Product law
$\log_{10}10 + \log_{10}10 = 1 + 1$
$= 2$
$= \log_{10}100$
$= \log_{10}(10 \times 10)$
Quotient law
$\log_{10}10 - \log_{10}10 = 1 - 1$
$= 0$
$= \log_{10}1$
$= \log_{10}\left(\frac{10}{10}\right)$
Power law
$\log_{10}10^2 = \log_{10}100$
$= 2$
$= 2\log_{10}10$

Mid-Chapter Review, p. 479

1. **a)** $\log_5 y = x$ **c)** $\log x = y$
b) $\log_{\frac{1}{3}} y = x$ **d)** $\log_p m = q$

2. **a)** $3^y = x$ **c)** $10^k = m$
b) $10^y = x$ **d)** $s^t = r$

3. **a)** vertical stretch by a factor of 2, vertical translation 4 units down
b) reflection in the x-axis, horizontal compression by a factor of $\frac{1}{3}$
c) vertical compression by a factor of $\frac{1}{4}$, horizontal stretch by a factor of 4
d) horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 2 units to the right
e) horizontal translation 5 units to the left, vertical translation 1 unit up
f) vertical stretch by a factor of 5, reflection in the y-axis, vertical translation 3 units down

4. **a)** $y = -4\log_3 x$
b) $y = \log_3(x + 3) + 1$
c) $y = \frac{2}{3}\log_3\left(\frac{1}{2}x\right)$
d) $y = 3\log_3[-(x - 1)]$

5. **a)** $(9, -8)$
b) $(6, 3)$
c) $\left(18, \frac{4}{3}\right)$
d) $(-8, 6)$

6. It is vertically stretched by a factor of 2 and vertically shifted up 2.

7. **a)** 4 **c)** 0
b) -2 **d)** -3

8. **a)** 0.602 **c)** 2.130
b) 1.653 **d)** 2.477

9. **a)** $x \doteq 4.392$ **c)** $x \doteq 2.543$
b) $x \doteq 2.959$ **d)** $x \doteq 2.450$

10. **a)** $\log 28$ **c)** $\log_3\frac{22}{3}$
b) $\log 2.5$ **d)** $\log_p q^2$

11. **a)** 1 **d)** -3
b) 2 **e)** $\frac{2}{3}$
c) 2 **f)** 3.5

12. Compared with the graph of $y = \log x$, the graph of $y = \log x^3$ is vertically stretched by a factor of 3.

13. **a)** 4.82 **d)** 1.69
b) 1.35 **e)** 3.82
c) 0.80 **f)** 3.49

Lesson 8.5, pp. 485–486

1. **a)** 4 **d)** $\frac{13}{9}$
b) 1 **e)** $-\frac{1}{3}$
c) $\frac{11}{4}$ **f)** -1

2. **a)** 4.088 **d)** 4.092
b) 3.037 **e)** -0.431
c) 1 **f)** 5.695

3. **a)** 5 **d)** $\frac{3}{5}$
b) 3 **e)** -2
c) 1.5 **f)** $-\frac{1}{2}$

4. **a)** 4.68 h **c)** 16 h
b) 12.68 h **d)** 31.26 h

5. **a)** 1.75 **d)** -4
b) $\frac{2}{3}$ **e)** 2
c) -4.75 **f)** 2

6. **a)** 9.12 years
b) 13.5 years
c) 16.44 quarters or 4.1 years
d) 477.9 weeks or 9.2 years

7. 13 quarter hours or 3.25 h

8. **a)** 2.5 **d)** 3
b) 6 **e)** 1
c) 5 **f)** 0

9. **a)** Solve using logarithms. Both sides can be divided by 225, leaving only a term with a variable in the exponent on the left. This can be solved using logarithms.
b) Solve by factoring out a power of 3 and then simplifying. Logarithms may still be necessary in a situation like this, but the factoring must be done first because logarithms cannot be used on the equation in its current form.

10. **a)** 1.849 **c)** 3.606
b) 2.931 **d)** 5.734

11. **a)** $I_f = I_o(0.95)^t$, where I_f is the final intensity, I_o is the original intensity, and t is the thickness
b) 10 mm

12. 1; 0.631

13. $a^y = x$, so $\log a^y = \log x$; $y\log a = \log x$; $y = \frac{\log x}{\log a}$
A graphing calculator does not allow logarithms of base 5 to be entered directly. However, $y = \log_5 x$ can be entered for graphing, as $y = \frac{\log x}{\log 5}$.

14. **a)** $x = 2.5$
b) $x = 5$ or $x = 4$
c) $x = -2.45$

15. Let $\log_a 2 = x$. Then $a^x = 2$. $(a^x)^3 = 2^3$, or $a^{3x} = 8$. Since $\log_a 2 = \log_b 8$, $\log_b 8 = x$. So $b^x = 8$. Since each equation is equal to 8, $a^{3x} = b^x$ and $a^3 = b$.

16. $x = -0.737$; $y = 0.279$

17. **a)** $x = -1.60$
b) $x = -4.86$
c) $x = -0.42$

18. ± 1.82

Lesson 8.6, pp. 491–492

1. **a)** 25 **d)** 15
b) 81 **e)** 3
c) 8 **f)** $\sqrt{3}$

2. **a)** 5 **d)** 200.4
b) $\frac{1}{36}$ **e)** 5
c) 13 **f)** 20

3. 201.43

4. **a)** 9 **d)** 10 000
b) $\sqrt{5}$ **e)** -3
c) $\frac{25}{3}$ **f)** 4

5. **a)** $\frac{8}{3}$ **d)** 32
b) $\frac{10}{3}$ **e)** 3
c) $\frac{25}{6}$ **f)** 8.1

6. $x = 9$ or $x = -4$
Restrictions: $x > 5$ ($x - 5$ must be positive) so $x = 9$

7. **a)** $x = 6$ **d)** $x = 2.5$
b) $x = 3$ **e)** $x = 3$
c) $x = \frac{6}{5}$ **f)** $x = 16$

8. **a)** Use the rules of logarithms to obtain $\log_9 20 = \log_9 x$. Then, because both sides of the equation have the same base, $20 = x$.
b) Use the rules of logarithms to obtain $\log\frac{x}{2} = 3$. Then use the definition of a logarithm to obtain $10^3 = \frac{x}{2}$; $1000 = \frac{x}{2}$; $2000 = x$.

c) Use the rules of logarithms to obtain $\log x = \log 64$. Then, because both sides of the equation have the same base, $x = 64$.

9. **a)** 10^{-7}
b) $10^{-3.6}$

10. $x = 2.5$ or $x = 2$

11. **a)** $x = 0.80$ **c)** $x = 3.16$
b) $x = -6.91$ **d)** $x = 0.34$

12. $x = 4.83$

13. $\log_3(-8) = x$; $3^x = -8$; Raising positive 3 to any power produces a positive value. If $x \geq 1$, then $3^x \geq 3$. If $0 \leq x < 1$, then $1 \leq x < 3$. If $x < 0$, then $0 < x < 1$.

14. **a)** $x > 3$
b) If x is 3, we are trying to take the logarithm of 0. If x is less than 3, we are trying to take the logarithm of a negative number.

15. $\frac{1}{2}(\log x + \log y) = \frac{1}{2}\log xy = \log \sqrt{xy}$ so $\frac{x+y}{5} = \sqrt{xy}$ and $x + y = 5\sqrt{xy}$. Squaring both sides gives $(x + y)^2 = 25xy$. Expanding gives $x^2 + 2xy + y = 25xy$; therefore, $x + y = 23xy$.

16. $x = 3$ or $x = 2$

17. 1 and 16, 2 and 8, 4 and 4, 8 and 2, and 16 and 1

18. $x = 4, y = 4.58$

19. **a)** $x = 3$
b) $x = 16$

20. $x = -1.75, y = -2.25$

Lesson 8.7, pp. 499–501

1. First earthquake: $5.2 = \log x$; $10^{5.2} = 158\,489$
Second earthquake; $6 = \log x$; $10^6 = 1\,000\,000$
Second earthquake is 6.3 times stronger than the first.

2. 7.2

3. 60 dB

4. 7.9 times

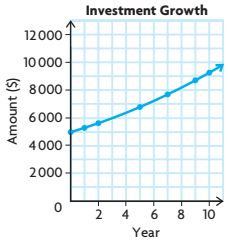
5. **a)** 0.000 000 001
b) 0.000 000 251
c) 0.000 000 016
d) 0.000 000 000 000 1

6. **a)** 3.49
b) 3.52
c) 4.35
d) 2.30

7. **a)** 7
b) Tap water is more acidic than distilled water as it has a lower pH than distilled water (pH 7).

8. 7.98 times

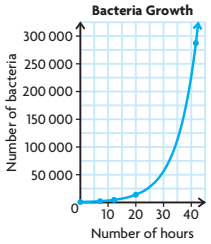
9. **a)** $y = 5000(1.0642)^t$



b) 6.42%
c) 11.14 years

10. 2.90 m

11. **a)** $y = 850(1.15)^x$



b) 4.9 h

12. **a)** 1.22, 1.43, 1.69, 2.00, 2.18, 2.35
b) 1.81
c) $w = 5.061\,88(1.061\,8)^t$
d) $w = 5.061\,88(1.061\,8)^t$
e) 11.5 °C

13. 33 cycles

14. 7.4 years

15. 26.2 days

16. Answers may vary. For example: (1) Tom invested $2000 in an account that accrued interest, compounded annually, at a rate of 6%. How long will it take for Tom's investment to triple? (2) Indira invested $5000 in a stock that made her $75 every month. How long will it take her investment to triple?
The first problem could be modelled using an exponential function. Solving this problem would require the use of logarithms. The second problem could be modelled using a linear equation. Solving the second problem would not require the use of logarithms.

17. 73 dB

18. **a)** $C = P(1.038)^t$
b) $580.80
c) $33.07

Lesson 8.8, pp. 507–508

1. **a)** -7.375
b) -23.25
c) -2

2. The instantaneous rate of decline was greatest in year 1. The negative change from year 1 to year 2 was 50, which is greater than the negative change in any other two-year period.

3. **a)** -12.378
b) -4.867
c) -1.914

4. **a)** $A(t) = 6000(1.075)^t$
b) 894.35
c) 461.25

5. **a)** **i)** 61.80
ii) 67.65
iii) 79.08
b) The rate of change is not constant because the value of the account each year is determined by adding a percent of the previous year's value.

6. **a)** 20.40 g
b) -0.111 g/h

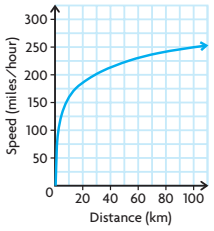
7. **a)** 1.59 g/day
b) $y = 0.0017(1.7698)^x$, where x is the number of days after the egg is laid
c) **i)** 0.0095 g/day
ii) 0.917 g/day
iii) 88.25 g/day
d) 14.3 days

8. **a)** 3.81 years
b) 9.5%/year

9. **a)** $y = 12\,000(0.982)^t$
b) -181.7 people/year
c) -109 people/year

10. Both functions approach a horizontal asymptote. Each change in x yields a smaller and smaller change in y. Therefore, the instantaneous rate of change grows increasingly small, toward 0, as x increases.

11. **a)**



b) 1.03 miles/hour/hour
c) 4.03 miles/hour/hour and 0.403 miles/hour/hour
d) The rate at which the wind changes during shorter distances is much greater than the rate at which the wind changes at farther distances. As the distance increases, the rate of change approaches 0.

12. To calculate the instantaneous rate of change for a given point, use the exponential function to calculate the values of y that approach the given value of x. Do this for values on either side of the given

value of x. Determine the average rate of change for these values of x and y. When the average rate of change has stabilized to a constant value, this is the instantaneous rate of change.

13. **a)** and **b)** Only a and k affect the instantaneous rate of change. Increases in the absolute value of either parameter tend to increase the instantaneous rate of change.

Chapter Review, pp. 510–511

1. **a)** $y = \log_4 x$ **c)** $y = \log_{\frac{1}{4}} x$
b) $y = \log_a x$ **d)** $m = \log_p q$

2. **a)** vertical stretch by a factor of 3, reflection in the x-axis, horizontal compression by a factor of $\frac{1}{2}$
b) horizontal translation 5 units to the right, vertical translation 2 units up
c) vertical compression by a factor of $\frac{1}{2}$, horizontal compression by a factor of $\frac{1}{5}$
d) horizontal stretch by a factor of 3, reflection in the y-axis, vertical shift 3 units down

3. **a)** $y = \frac{2}{5}\log x - 3$
b) $y = -\log\left[\frac{1}{2}(x - 3)\right]$
c) $y = 5\log(-2x)$
d) $y = \log(-x - 4) - 2$

4. Compared to $y = \log x$, $y = 3\log(x - 1) + 2$ is vertically stretched by a factor of 3, horizontally translated 1 unit to the right, and vertically translated 2 units up.

5. **a)** 3 **c)** 0
b) -2 **d)** -4

6. **a)** 3.615 **c)** 2.829
b) -1.661 **d)** 2.690

7. **a)** $\log 55$ **c)** $\log_5 4$
b) $\log 5$ **d)** $\log 128$

8. **a)** 1 **c)** $\frac{2}{3}$
b) 2 **d)** 3

9. It is shifted 4 units up.

10. **a)** 5 **c)** -2
b) 3.75 **d)** -0.2

11. **a)** 2.432 **c)** 2.553
b) 3.237 **d)** 4.799

12. **a)** 0.79; 0.5
b) -0.43

13. 5.45 days

14. **a)** 63 **c)** 9
b) $\frac{10\,000}{3}$ **d)** 1.5

15. **a)** 1 **c)** 3
b) 5 **d)** $\pm\sqrt{10\,001}$

16. 10^{-2} W/m^2

17. $10^{-3.8}$ W/m^2

18. 5 times

19. 3.9 times

20. $\frac{10^{4.7}}{10^{2.3}} = 251.2$
$\frac{10^{12.5}}{10^{10.1}} = 251.2$
The relative change in each case is the same. Each change produces a solution with concentration 251.2 times the orignial solution.

21. Yes; $y = 3(2.25^x)$

22. 17.8 years

23. **a)** 8671 people per year
b) 7114; The rate of growth for the first 30 years is slower than the rate of growth for the entire period.
c) $y = 134\,322(1.03^x)$, where x is the number of years after 1950
d) **i)** 7171 people per year
ii) 12 950 people per year

24. **a)** exponential; $y = 23(1.17^x)$, where x is the number of years since 1998
b) 331 808
c) Answers may vary. For example, I assumed that the rate of growth would be the same through 2015. This is not reasonable. As more people buy the players, there will be fewer people remaining to buy them, or newer technology may replace them.
d) about 5300 DVD players per year
e) about 4950 DVD players per year
f) Answers may vary. For example, the prediction in part e) makes sense because the prediction is for a year covered by the data given. The prediction made in part b) does not make sense because the prediction is for a year that is beyond the data given, and conditions may change, making the model invalid.

Chapter Self-Test, p. 512

1. **a)** $x = 4^y$; $\log_4 x = y$
b) $y = 6^x$; $\log_6 y = x$

2. **a)** horizontal compression by a factor of $\frac{1}{2}$, horizontal translation 4 units to the right, vertical translation 3 units up
b) vertical compression by a factor of $\frac{1}{2}$, reflection in the x-axis, horizontal translation 5 units to the left, vertical translation 1 unit down

3. **a)** -2 **b)** 5

4. **a)** 2 **b)** 7

5. $\log_4 xy$

6. 7.85

7. **a)** 2 **b)** $1\frac{3}{4}$

8. **a)** 50 g
b) $A(t) = 100(0.5)^{\frac{t}{5730}}$
c) 1844 years
d) -0.015 g/year

9. **a)** 6 min
b) 97°

Chapter 9

Getting Started, p. 516

1. **a)** $f(-1) = 30$, $f(4) = 0$
b) $f(-1) = -2$, $f(4) = -5\frac{1}{3}$
c) $f(-1)$ is undefined, $f(4) \doteq 1.81$
d) $f(-1) = -20$, $f(4) = -0.625$

2. $D = \{x \in \mathbf{R} \mid x \neq 1\}$
$R = \{y \in \mathbf{R} \mid y \neq 2\}$
There is no minimum or maximum value;
the function is never increasing;
the function is decreasing from $(-\infty, 1)$ and $(1, \infty)$;
the function approaches $-\infty$ as x approaches 1 from the left and ∞ as x approaches 1 from the right;
vertical asymptote is $x = 1$;
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3. **a)** $y = 2|x - 3|$
b) $y = -\cos(2x)$
c) $y = \log_3(-x - 4) - 1$
d) $y = -\frac{4}{x} - 5$

4. **a)** $x = -1, \frac{1}{2}$, and 4
b) $x = -\frac{5}{3}$ or $x = 3$
c) $x = 5$ or $x = -2$
Cannot take the log of a negative number, so $x = 5$.
d) $x = -\frac{3}{4}$
e) $x = -3$
f) $\sin x = \frac{3}{2}$ or $\sin x = -1$. Since $\sin x$ cannot be greater than 1, the first equation does not give a solution; $x = 270°$

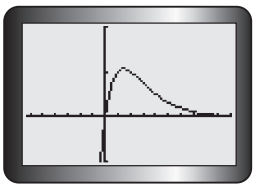
5. **a)** $(-\infty, -4) \cup (2, 3)$
b) $\left(-2, \frac{3}{2}\right) \cup [4, \infty)$

6. **a)** odd **c)** even
b) neither **d)** neither

7. Polynomial, logarithmic, and exponential functions are continuous. Rational and trigonometric functions are sometimes continuous and sometimes not.

Lesson 9.1, p. 520

1. Answers may vary. For example, the graph of $y = \left(\left(\frac{1}{2}\right)^x\right)(2x)$ is



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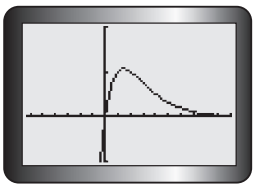
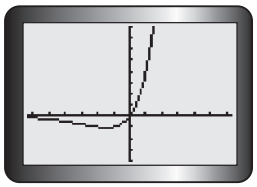
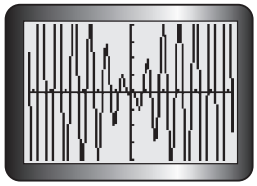


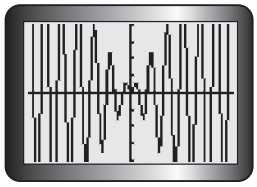
2. **a)** Answers may vary. For example, $y = (2^x)(2x)$;



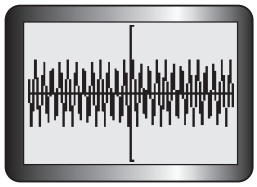
b) Answers may vary. For example, $y = (2x)(\cos(2\pi x))$;



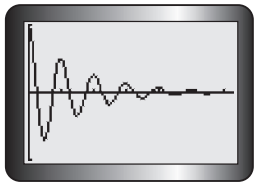
c) Answers may vary. For example, $y = (2x)(\sin(2\pi x))$;



d) Answers may vary. For example, $y = (\sin 2\pi x)(\cos 2\pi x)$;



e) Answers may vary. For example, $y = \left(\frac{1}{2}\right)^x(\cos 2\pi x)$, where $0 \le x \le 2\pi$;



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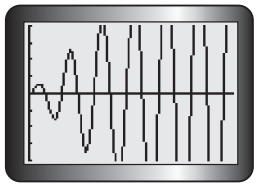
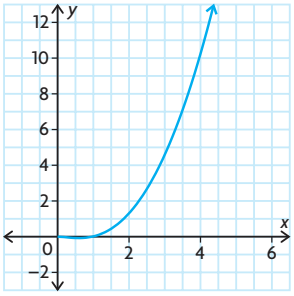


3. Answers will vary. For example,
$y = x^2$
$y = \log x$
The product will be $y = x^2 \log x$.



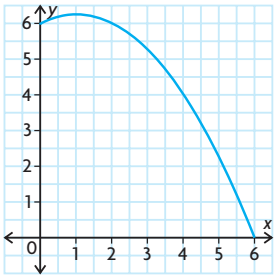
Lesson 9.2, pp. 528–530

1. **a)** $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
b) $\{(-4, 6), (-2, 5), (1, 5), (4, 10)\}$
c) $\{(-4, 2), (-2, 3), (1, 1), (4, 2)\}$
d) $\{(-4, -2), (-2, -3), (1, -1), (4, -2)\}$
e) $\{(-4, 8), (-2, 8), (1, 6), (3, 10), (4, 12)\}$
f) $\{(-4, 0), (-2, 0), (0, 0), (1, 0), (2, 0), (4, 0)\}$

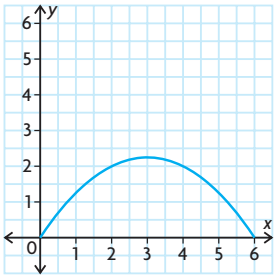
2. **a)** 10
b) 2; $(f + g)(x)$ is undefined at $x = 2$ because $g(x)$ is undefined at $x = 2$.
c) $\{x \in \mathbf{R} \mid x \ne 2\}$

3. $\{x \in \mathbf{R} \mid -1 \le x < 1\}$

4. Graph of $f + g$:



Graph of $f - g$:

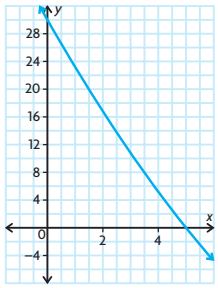


5. **a)** $f + g = |x| + x$
b) The function is neither even nor odd.

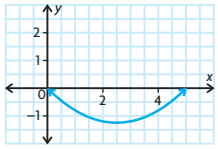
6. **a)** $\{(-6, 7), (-3, 10)\}$
b) $\{(-6, 7), (-3, 10)\}$
c) $\{(-6, -5), (-3, 4)\}$
d) $\{(-6, 5), (-3, -4)\}$
e) $\{(-9, 0), (-8, 0), (-6, 0), (-3, 0), (-1, 0), (0, 0)\}$
f) $\{(-7, 14), (-6, 12), (-5, 10), (-4, 8), (-3, 6)\}$

7. **a)** $\frac{2(2x + 1)}{3x^2 - 2x - 8}$
b) $\left\{x \in \mathbf{R} \mid x \ne -\frac{4}{3} \text{ or } 2\right\}$
c) $\frac{17}{84}$
d) $-\frac{11}{84}$

8. The graph of $(f + g)(x)$:



The graph of $(f - g)(x)$:



9. **a)** $f(x) + g(x) = 2^x + x^3$
The function is not symmetric.
The function is always increasing.
zero at $x = -0.8262$
no maximum or minimum
period: N/A
The domain is all real numbers. The range is all real numbers.
$f(x) - g(x) = 2^x - x^3$
The function is not symmetric.
The function is always decreasing.
zero at $x = 1.3735$
no maximum or minimum
period: N/A
The domain is all real numbers. The range is all real numbers.
b) $f(x) + g(x) = \cos(2\pi x) + x^4$
The function is symmetric across the line $x = 0$.
The function is decreasing from $-\infty$ to -0.4882 and 0 to 0.4882 and increasing from -0.4882 to 0 and 0.4882 to ∞.
zeros at $x = -0.7092, -0.2506$, 0.2506, 0.7092

relative maximum at $x = 0$ and relative minimums at $x = -0.4882$ and $x = 0.4882$
period: N/A
The domain is all real numbers. The range is all real numbers greater than -0.1308.

$f(x) - g(x) = \cos(2\pi x) - x^4$
The function is symmetric across the line $x = 0$.
The function is increasing from $-\infty$ to -0.9180 and -0.5138 to 0 and 0.5138 to 0.9180; decreasing from -0.9180 to -0.5138 and 0 to 0.5138 and 0.9180 to ∞.
zeros at $x = -1, -0.8278, -0.2494$, 0.2494, 0.8278, 1
relative maxima at -0.9180, 0, and 0.9180; relative minima at -0.5138 and 0.5138
period: N/A
The domain is all real numbers. The range is all real numbers less than 1.

c) $f(x) + g(x) = \log(x) + 2x$
The function is not symmetric.
The function is increasing from 0 to ∞.
no zeros
no maximum or minimum
period: N/A
The domain is all real numbers greater than 0. The range is all real numbers.

$f(x) - g(x) = \log(x) - 2x$
The function is not symmetric.
The function is increasing from 0 to approximately 0.2 and decreasing from approximately 0.2 to ∞.
no zeros
maximum at $x \doteq 0.2$
period: N/A
The domain is all real numbers greater than 0. The range is all real numbers less than or equal to approximately -1.1.

d) $f(x) + g(x) = \sin(2\pi x) + 2\sin(\pi x)$
The function is symmetric about the origin.
The function is increasing from $-0.33 + 2k$ to $0.33 + 2k$ and decreasing from $0.33 + 2k$ to $1.67 + 2k$.
zero at k
minimum at $x = -0.33 + 2k$
maximum at $x = 0.33 + 2k$
period: 2
The domain is all real numbers. The range is all real numbers between -2.598 and 2.598.

$f(x) - g(x) = \sin(2\pi x) - 2\sin(\pi x)$
The function is symmetric about the origin, increasing from $0.67 + 2k$ to $1.33 + 2k$ and decreasing from $-0.67 + 2k$ to $0.67 + 2k$
zero at k
minimum at $0.67 + 2k$ and maximum at $1.33 + 2k$
period: 2
The domain is all real numbers.
The range is all real numbers between -2.598 to 2.598.

e) $f(x) + g(x) = \sin(2\pi x) + \frac{1}{x}$
The function is not symmetric.
The function is increasing and decreasing at irregular intervals.
The zeros are changing at irregular intervals.
The maximums and minimums are changing at irregular intervals.
period: N/A
The domain is all real numbers except 0.
The range is all real numbers.

$f(x) - g(x) = \sin(2\pi x) - \frac{1}{x}$
The function is not symmetric.
The function is increasing and decreasing at irregular intervals.
The zeros are changing at irregular intervals.
The maximums and minimums are changing at irregular intervals.
period: N/A
The domain is all real numbers except 0.
The range is all real numbers.

f) $f(x) + g(x) = \sqrt{x-2} + \frac{1}{x-2}$
The function is not symmetric.
The function is increasing from 3.5874 to ∞ and decreasing from 2 to 3.5874.
zeros: none
minimum at $x = 3.5874$
period: N/A
The domain is all real numbers greater than 2. The range is all real numbers greater than 1.8899.

$f(x) - g(x) = \sqrt{x-2} - \frac{1}{x-2}$
The function is not symmetric.
The function is increasing from 2 to ∞.
zero at $x = 3$
no maximum or minimum
period: N/A
The domain is all real numbers greater than 2. The range is all real numbers.

10. a) The sum of two even functions will be even because replacing x with $-x$ will still result in the original function.

b) The sum of two odd functions will be odd because replacing x with $-x$ will still result in the opposite of the original function.

c) The sum of an even and an odd function will result in neither an even nor an odd function because replacing x with $-x$ will not result in the same function or in the opposite of the function.

11. a) $R(t) = 5000 - 25t - 1000\cos\left(\frac{\pi}{6}t\right)$; it is neither odd nor even; it is increasing during the first 6 months of each year and decreasing during the last 6 months of each year; it has one zero, which is the point at which the deer population has become extinct; it has a maximum value of 3850 and a minimum value of 0, so its range is $\{R(t) \in \mathbf{R} \mid 0 \leq R(t) \leq 3850\}$.

b) after about 167 months, or 13 years and 11 months

12. The stopping distance can be defined by the function $s(x) = 0.006x^2 + 0.21x$. If the vehicle is travelling at 90 km/h, the stopping distance is 67.5 m.

13. $f(x) = \sin(\pi x)$; $g(x) = \cos(\pi x)$

14. The function is neither even nor odd; it is not symmetrical with respect to the y-axis or with respect to the origin; it extends from the third quadrant to the first quadrant; it has a turning point between $-n$ and 0 and another turning point at 0; it has zeros at $-n$ and 0; it has no maximum or minimum values; it is increasing when $x \in (-\infty, -n)$ and when $x \in (0, \infty)$; when $x \in (-n, 0)$, it increases, has a turning point, and then decreases; its domain is $\{x \in \mathbf{R}\}$, and its range is $\{y \in \mathbf{R}\}$.

15. a) $f(x) = 0$; $g(x) = 0$

b) $f(x) = x^2$; $g(x) = x^2$

c) $f(x) = \frac{1}{x-2}$; $g(x) = \frac{1}{x-2} + 2$.

16. $m = 2, n = 3$

Lesson 9.3, pp. 537–539

1. a) $\{(0, -2), (1, -10), (2, 21), (3, 60)\}$

b) $\{(0, 12), (2, -20)\}$

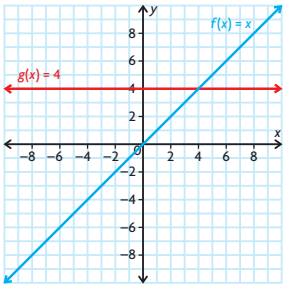
c) $4x$

d) $2x^2$

e) $x^3 - 3x + 2$

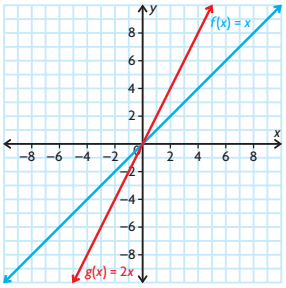
f) $2^x\sqrt{x-2}$

2. a) 1(c):

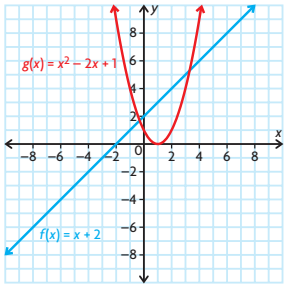


Answers

1(d):



1(e):



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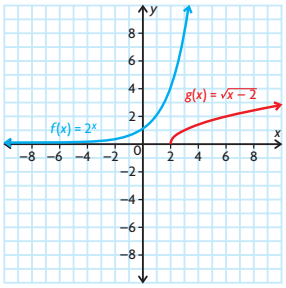
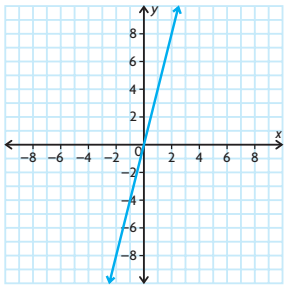
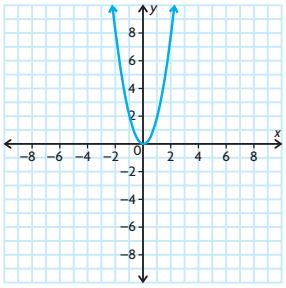


b) 1(c): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(d): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(e): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R}\}$
1(f): f: $\{x \in \mathbf{R}\}$; g: $\{x \in \mathbf{R} \mid x \geq 2\}$

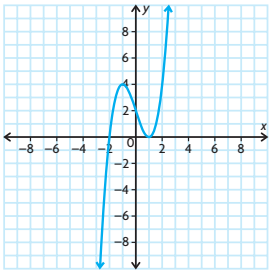
c) 1(c):



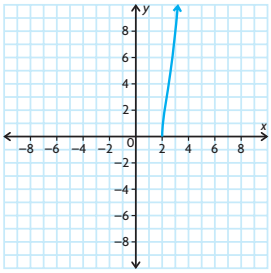
1(d):



1(e):



1(f):



d) 1(c): $\{x \in \mathbf{R}\}$
1(d): $\{x \in \mathbf{R}\}$
1(e): $\{x \in \mathbf{R}\}$
1(f): $\{x \in \mathbf{R} \mid x \geq 2\}$

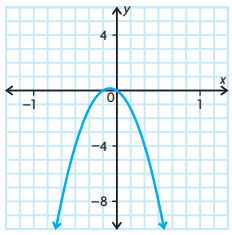
3. $\{x \in \mathbf{R} \mid -1 \leq x \leq 1\}$

4. **a)** $x^2 - 49$
b) $x + 10$
c) $7x^3 - 63x^2$
d) $-16x^2 - 56x - 49$
e) $\dfrac{2 \sin x}{x - 1}$
f) $2^x \log(x + 4)$

5. 4(a): D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R} \mid y \geq -49\}$
4(b): D $= \{x \in \mathbf{R} \mid x \geq -10\}$;
R $= \{y \in \mathbf{R} \mid y \geq 0\}$
4(c): D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R}\}$
4(d): D $= \{x \in \mathbf{R}\}$; R $= \{y \in \mathbf{R} \mid y \leq 0\}$
4(e): D $= \{x \in \mathbf{R} \mid x \neq -1\}$; R $= \{y \in \mathbf{R}\}$
4(f): D $= \{x \in \mathbf{R} \mid x > -4\}$;
R $= \{y \in \mathbf{R} \mid y \geq 0\}$

6. 4(a): The function is symmetric about the line $x = 0$.
The function is increasing from 0 to ∞.
The function is decreasing from $-\infty$ to 0.
zeros at $x = -7, 7$
The minimum is at $x = 0$.
period: N/A
4(b): The function is not symmetric.
The function is increasing from -10 to ∞.
zero at $x = -10$
The minimum is at $x = -10$.
period: N/A
4(c): The function is not symmetric.
The function is increasing from $-\infty$ to 0 and from 6 to ∞.
zeros at $x = 0, 9$
The relative minimum is at $x = -6$. The relative maximum is at $x = 0$.
period: N/A
4(d): The function is symmetric about the line $x = -1.75$.
The function is increasing from $-\infty$ to -1.75 and is decreasing from -1.75 to ∞.
zero at $x = -1.75$
The maximum is at $x = -1.75$.
period: N/A
4(e): The function is not symmetric.
The function is increasing from $-\infty$ to 0 and from 6 to ∞.
zeros at $x = 0, 9$
The relative minima are at $x = -4.5336$ and 4.4286. The relative maximum is at $x = -1.1323$.
period: N/A
4(f): The function is not symmetric.
The function is increasing from -4 to ∞.
zeros: none
maximum/minimum: none
period: N/A

7.



8. **a)** $\left\{x \in \mathbf{R} \middle| x \neq -2, 7, \dfrac{\pi}{2}, \text{ or } \dfrac{3\pi}{2}\right\}$
b) $\{x \in \mathbf{R} \mid x > 8\}$
c) $\{x \in \mathbf{R} \mid x \geq -81 \text{ and } x \neq 0, \pi, \text{ or } 2\pi\}$
d) $\{x \in \mathbf{R} \mid x \leq -1 \text{ or } x \geq 1, \text{ and } x \neq -3\}$

9. $(f \times p)(t)$ represents the total energy consumption in a particular country at time t

10. **a)** $R(x) = (20\,000 - 750x)(25 + x)$ or $R(x) = 500\,000 + 1250x - 750x^2$, where x is the increase in the admission fee in dollars

b) Yes, it's the product of the function $P(x) = 20\,000 - 750x$, which represents the number of daily visitors, and $F(x) = 25 + x$, which represents the admission fee.

c) \$25.83

11. $m(t) = ((0.9)^t)(650 + 300t)$
The amount of contaminated material is at its greatest after about 7.3 s.

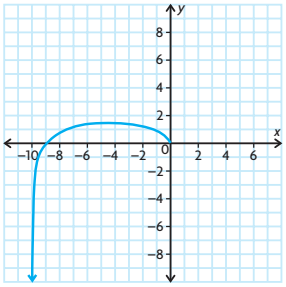
12. The statement is false. If $f(x)$ and $g(x)$ are odd functions, then their product will always be an even function. When you multiply a function that has an odd degree with another function that has an odd degree, you add the exponents, and when you add two odd numbers together, you get an even number.

13. $f(x) = 3x^2 + 2x + 5$ and $g(x) = 2x^2 - 4x - 2$

14. **a)** $(f \times g)(x) = \sqrt{-x}\log(x + 10)$
The domain is $\{x \in \mathbf{R} \mid -10 < x \leq 0\}$.

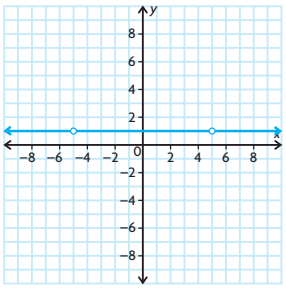
b) One strategy is to create a table of values for $f(x)$ and $g(x)$ and to multiply the corresponding y-values together. The resulting values could then be graphed. Another strategy is to graph $f(x)$ and $g(x)$ and to then create a graph for $(f \times g)(x)$ based on these two graphs. The first strategy is probably better than the second strategy, since the y-values for $f(x)$ and $g(x)$ will not be round numbers and will not be easily discernable from the graphs of $f(x)$ and $g(x)$.

c)



15. **a)** $f(x) \times \dfrac{1}{f(x)} = 1$

b) $\{x \in \mathbf{R} \mid x \neq -5 \text{ or } 5\}$



c) The range will always be 1. If f is of odd degree, there will always be at least one value that makes the product undefined and which is excluded from the domain. If f is of even degree, there may be no values that are excluded from the domain.

16. **a)** $f(x) = 2^x$
$g(x) = x^2 + 1$
$(f \times g)(x) = 2^x(x^2 + 1)$

b) $f(x) = x$
$g(x) = \sin(2\pi x)$
$(f \times g)(x) = x\sin(2\pi x)$

17. **a)** $f(x) = (2x + 9)$
$g(x) = (2x - 9)$

b) $f(x) = (2\sin x + 3)$
$g(x) = (4\sin^2 x - 6\sin x + 9)$

c) $f(x) = x^{\frac{1}{2}}$
$g(x) = (4x^5 - 3x^3 + 1)$

d) $f(x) = \dfrac{1}{2x + 1}$
$g(x) = 6x - 5$

Lesson 9.4, p. 542

1. **a)** $(f \div g)(x) = \dfrac{5}{x}, x \neq 0$

b) $(f \div g)(x) = \dfrac{4x}{2x - 1}, x \neq \dfrac{1}{2}$

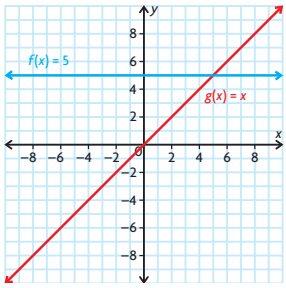
c) $(f \div g)(x) = \dfrac{4x}{x^2 + 4}$

d) $(f \div g)(x) = \dfrac{(x + 2)\left(\sqrt{x - 2}\right)}{x - 2}, x > 2$

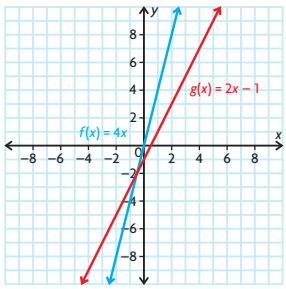
e) $(f \div g)(x) = \dfrac{8}{1 + \left(\frac{1}{2}\right)^x}$

f) $(f \div g)(x) = \dfrac{x^2}{\log(x)}, x > 0$

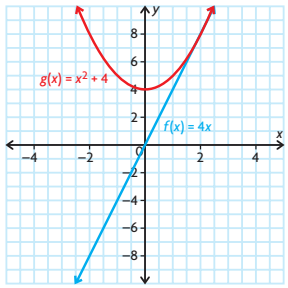
2. **a)** 1(a):



1(b):



1(c):



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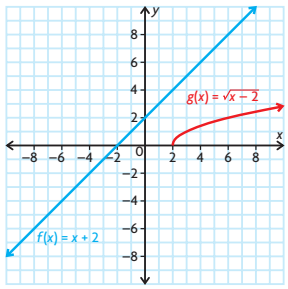

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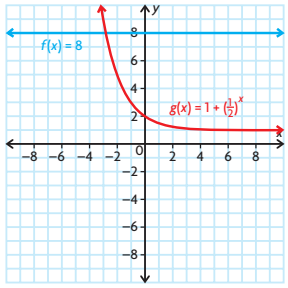

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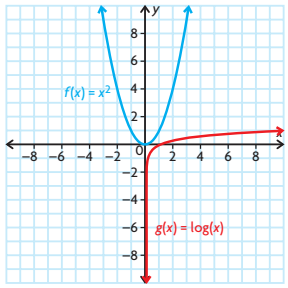
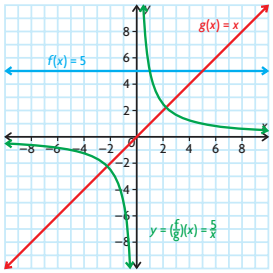
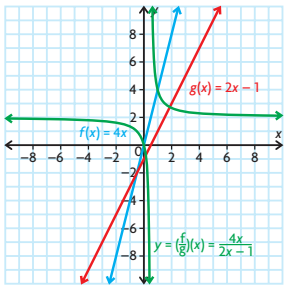


b) 1(a): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(b): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(c): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(d): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R} \mid x \geq 2\}$
1(e): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R}\}$
1(f): domain of f: $\{x \in \mathbf{R}\}$;
domain of g: $\{x \in \mathbf{R} \mid x > 0\}$

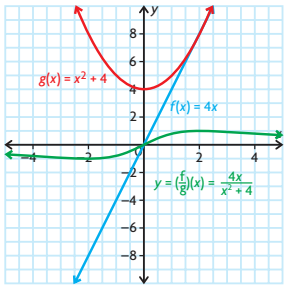
c) 1(a):



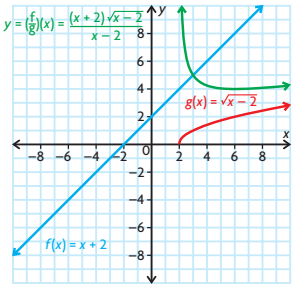
1(b):



1(c):



1(d):



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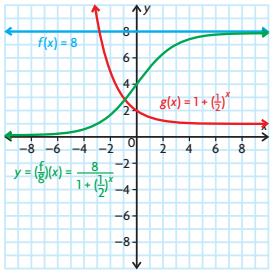

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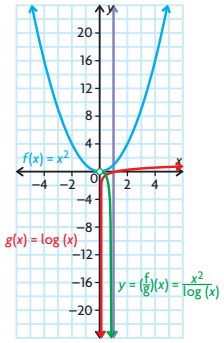


d) 1(a): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x \neq 0\}$
1(b): domain of $(f \div g)$: $\left\{x \in \mathbf{R} \mid x \neq \frac{1}{2}\right\}$
1(c): domain of $(f \div g)$: $\{x \in \mathbf{R}\}$
1(d): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x > 2\}$
1(e): domain of $(f \div g)$: $\{x \in \mathbf{R}\}$
1(f): domain of $(f \div g)$: $\{x \in \mathbf{R} \mid x > 0\}$

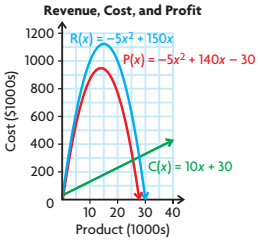
3. a) 2.798 cm/day
b) about 30 days
c) 6.848 cm/day
d) It slows down and eventually comes to zero. This is seen on the graph as it becomes horizontal at the top.

Mid-Chapter Review, p. 544

1. multiplication

2. a) $\{(-9, 2), (-6, -9), (0, 14)\}$
b) $\{(-9, 2), (-6, -9), (0, 14)\}$
c) $\{(-9, -6), (-6, 3), (0, -10)\}$
d) $\{(-9, 6), (-6, -3), (0, 10)\}$

3. a) $P(x) = -5x^2 + 140x - 30$
b)



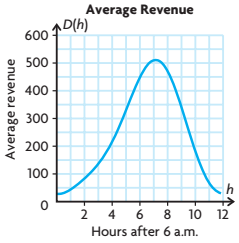
c) $738 750

4. a) $R(h) = 24.39h$
b) $N(h) = 24.97h$
c) $W(h) = 24.78h$
d) $S(h) = 25.36h$
e) $317

5. a) $(f \times g)(x) = x^2 + x + \frac{1}{4}$
$D = \{x \in \mathbf{R}\}$
b) $(f \times g)(x) = \sin(3x)(\sqrt{x - 10})$
$D = \{x \in \mathbf{R} \mid x \geq 10\}$
c) $(f \times g)(x) = \frac{22x^3}{x + 5}$
$D = \{x \in \mathbf{R} \mid x \neq -5\}$
d) $(f \times g)(x) = 8100x^2 - 1$
$D = \{x \in \mathbf{R}\}$

6. a) $R(h) = 90 \cos\left(\frac{\pi}{6}h\right) \sin\left(\frac{\pi}{6}h\right) - 102 \sin\left(\frac{\pi}{6}h\right) - 210 \cos\left(\frac{\pi}{6}h\right) + 238$

b)



c) about $470.30

7. a) $(f \div g)(x) = \frac{80}{x}$
$D = \{x \in \mathbf{R} \mid x \neq 0\}$
b) $(f \div g)(x) = \frac{10x^2}{x^2 - 3}$
$D = \{x \in \mathbf{R} \mid x \neq \pm\sqrt{3}\}$
c) $(f \div g)(x) = \frac{x + 8}{\sqrt{x - 8}}$
$D = \{x \in \mathbf{R} \mid x > 8\}$
d) $(f \div g)(x) = \frac{7x^2}{\log x}$
$D = \{x \in \mathbf{R} \mid x > 0\}$

8. csc x, sec x, cot x

Lesson 9.5, pp. 552–554

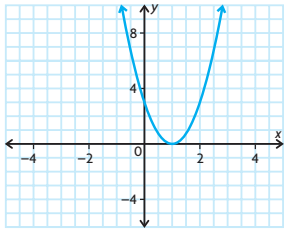
1. a) -1
b) -24
c) -129
d) $\frac{7}{16}$
e) 1
f) -8

2. a) 3
b) 5
c) 10
d) $(f \circ g)(0)$ is undefined.
e) 2
f) 4

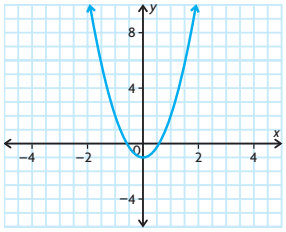
3. a) 5
b) 5
c) 4
d) $(f \circ f)(2)$ is undefined.

4. a) $C(d(5)) = 36$
It costs $36 to travel for 5 h.
b) $C(d(t))$ represents the relationship between the time driven and the cost of gasoline.

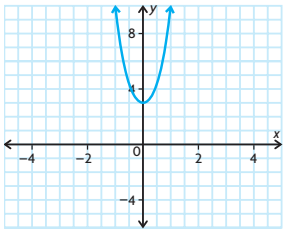
5. a) $f(g(x)) = 3x^2 - 6x + 3$
The domain is $\{x \in \mathbf{R}\}$.



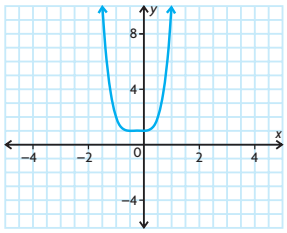
$g(f(x)) = 3x^2 - 1$
The domain is $\{x \in \mathbf{R}\}$.



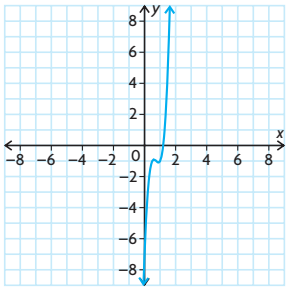
b) $f(g(x)) = 2x^4 + 5x^2 + 3$
The domain is $\{x \in \mathbf{R}\}$.



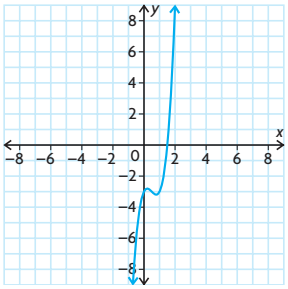
$g(f(x)) = 4x^4 + 4x^3 + x^2 + 1$
The domain is $\{x \in \mathbf{R}\}$.



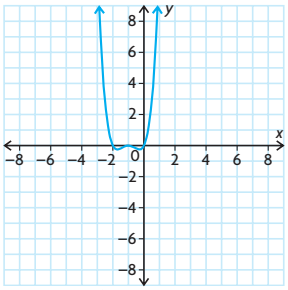
c) $f(g(x)) = 16x^3 - 36x^2 + 26x - 7$
The domain is $\{x \in \mathbf{R}\}$.



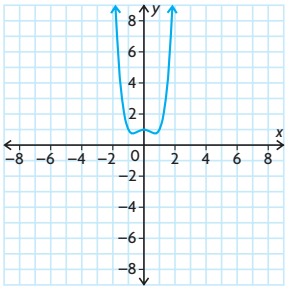
$g(f(x)) = 4x^3 - 6x^2 + 2x - 3$
The domain is $\{x \in \mathbf{R}\}$.



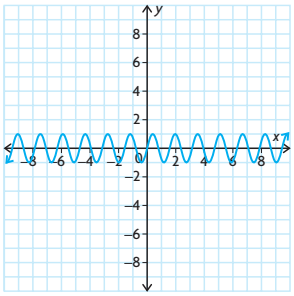
d) $f(g(x)) = x^4 + 4x^3 + 5x^2 + 2x$
The domain is $\{x \in \mathbf{R}\}$.



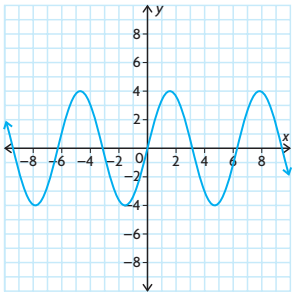
$g(f(x)) = x^4 - x^2 + 1$
The domain is $\{x \in \mathbf{R}\}$.



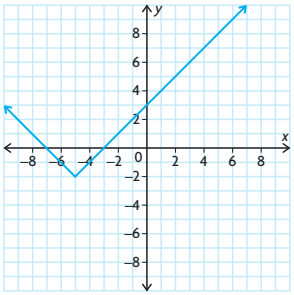
e) $f(g(x)) = \sin 4x$
The domain is $\{x \in \mathbf{R}\}$.



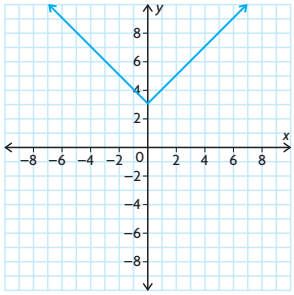
$g(f(x)) = 4 \sin x$
The domain is $\{x \in \mathbf{R}\}$.



f) $f(g(x)) = |x + 5| - 2$
The domain is $\{x \in \mathbf{R}\}$.



$g(f(x)) = |x| + 3$
The domain is $\{x \in \mathbf{R}\}$.



6. a) $f \circ g = 3\sqrt{x - 4}$
$D = \{x \in \mathbf{R} \mid x \geq 4\}$
$R = \{y \in \mathbf{R} \mid y \geq 0\}$
$g \circ f = \sqrt{3x - 4}$
$D = \left\{x \in \mathbf{R} \mid x \geq \frac{4}{3}\right\}$
$R = \{y \in \mathbf{R} \mid y \geq 0\}$

b) $f \circ g = \sqrt{3x + 1}$
$D = \left\{x \in \mathbf{R} \mid x \geq -\frac{1}{3}\right\}$
$R = \{y \in \mathbf{R} \mid y \geq 0\}$
$g \circ f = 3\sqrt{x} + 1$
$D = \{x \in \mathbf{R} \mid x \geq 0\}$
$R = \{y \in \mathbf{R} \mid y \geq 1\}$

c) $f \circ g = \sqrt{4 - x^4}$
D $= \{x \in \mathbf{R} \mid -\sqrt{2} \le x \le \sqrt{2}\}$
R $= \{y \in \mathbf{R} \mid y \ge 0\}$
$g \circ f = 4 - x^2$
D $= \{x \in \mathbf{R} \mid -2 \le x \le 2\}$
R $= \{y \in \mathbf{R} \mid 0 < y < 2\}$

d) $f \circ g = 2\sqrt{x - 1}$
D $= \{x \in \mathbf{R} \mid x \ge 1\}$
R $= \{y \in \mathbf{R} \mid y \ge 1\}$
$g \circ f = \sqrt{2^x - 1}$
D $= \{x \in \mathbf{R} \mid x \ge 0\}$
R $= \{y \in \mathbf{R} \mid y \ge 0\}$

e) $f \circ g = x$
D $= \{x \in \mathbf{R} \mid x > 0\}$
R $= \{y \in \mathbf{R}\}$
$g \circ f = x$
D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R}\}$

f) $f \circ g = \sin(5^{2x} + 1)$
D $= \{x \in \mathbf{R}\}$
R $= \{y \in \mathbf{R} \mid -1 \le y \le 1\}$
$g \circ f = 5^{2 \sin x} + 1$
D $= \{x \in \mathbf{R}\}$
R $= \left\{y \in \mathbf{R} \mid \frac{26}{25} \le y \le 26\right\}$

7. a) Answers may vary. For example, $f(x) = \sqrt{x}$ and $g(x) = x^2 + 6$

b) Answers may vary. For example, $f(x) = x^6$ and $g(x) = 5x - 8$

c) Answers may vary. For example, $f(x) = 2^x$ and $g(x) = 6x + 7$

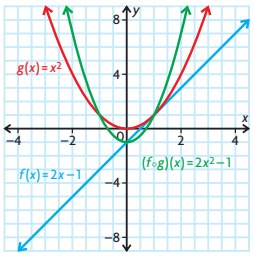
d) Answers may vary. For example, $f(x) = \frac{1}{x}$ and $g(x) = x^3 - 7x + 2$

e) Answers may vary. For example, $f(x) = \sin^2 x$ and $g(x) = 10x + 5$

f) Answers may vary. For example, $f(x) = \sqrt[3]{x}$ and $g(x) = (x + 4)^2$

8. a) $(f \circ g)(x) = 2x^2 - 1$

b)



c) It is compressed by a factor of 2 and translated down 1 unit.

9. a) $f(g(x)) = 6x + 3$
The slope of $g(x)$ has been multiplied by 2, and the y-intercept of $g(x)$ has been vertically translated 1 unit up.

b) $g(f(x)) = 6x - 1$
The slope of $f(x)$ has been multiplied by 3.

10. $D(p) = 780 + 31.96p$

11. $f(g(x)) = 0.06x$

12. a) $d(s) = \sqrt{16 + s^2}$; $s(t) = 560t$

b) $d(s(t)) = \sqrt{16 + 313\,600t^2}$, where t is the time in hours and $d(s(t))$ is the distance in kilometres

13. $c(v(t)) = \left(\frac{40 + 3t + t^2}{500} - 0.1\right)^2 + 0.15$;
The car is running most economically 2 h into the trip.

14. Graph A(k); $f(x)$ is vertically compressed by a factor of 0.5 and reflected in the x-axis. Graph B(b); $f(x)$ is translated 3 units to the left.
Graph C(d); $f(x)$ is horizontally compressed by a factor of $\frac{1}{2}$.
Graph D(1); $f(x)$ is translated 4 units down.
Graph E(g); $f(x)$ is translated 3 units up.
Graph F(c); $f(x)$ is reflected in the y-axis.

15. **Sum:** $y = f + g$
$f(x) = \frac{4}{x - 3}$; $g(x) = 1$
Product: $y = f \times g$
$f(x) = x - 3$; $g(x) = \frac{x + 1}{(x - 3)^2}$
Quotient: $y = f \div g$
$f(x) = 1 + x$; $g(x) = x - 3$
Composition: $y = f \circ g$
$f(x) = \frac{4}{x} + 1$; $g(x) = x - 3$

16. a) $f(k) = 27k - 14$

b) $f(k) = 2\sqrt{9k - 16} - 5$

Lesson 9.6, pp. 560–562

1. a) i) $x = \frac{1}{2}$, 2, or $\frac{7}{2}$

ii) $x = -1$ or 2

b) i) $\frac{1}{2} < x < 2$ or $x > \frac{7}{2}$

ii) $-1 < x < 2$

c) i) $x \le \frac{1}{2}$; $2 \le x \le \frac{7}{2}$

ii) $x \le -1$ or $x \ge 2$

d) i) $\frac{1}{2} \le x \le 2$ or $x \ge \frac{7}{2}$

ii) $-1 \le x \le 2$

2. a) $x \doteq 0.8$

b) $x = 0$ and 3.5

c) $x \doteq -2.4$

d) $x \doteq 0.7$

3. $x = -1.3$ or 1.8

4. $f(x) < g(x)$: $1.3 < x < 1.6$
$f(x) = g(x)$: $x = 0$ or 1.3
$f(x) > g(x)$: $0 < x < 1.3$ or $1.6 < x < 3$

5. a) $x \doteq 2.5$ **d)** $x \doteq -2.1$
b) $x \doteq 2.2$ **e)** $x = 10$
c) $x \doteq 1.8$ **f)** $x = 1$ or 3

6. a) $x = -1.81$ or 0.48

b) $x = -1.38$ or 1.6

c) $x = -1.38$ or 1.30

d) $x = -0.8$, 0, or 0.8

e) $x = 0.21$ or 0.74

f) $x = 0$, 0.18, 0.38, or 1

7. (0.7, −1.5)

8. They will be about the same in 2012.

9. a) $x \in (-0.57, 1)$

b) $x \in [0, 0.58]$

c) $x \in (-\infty, 0)$

d) $x \in (0.17, 0.83)$

e) $x \in (0.35, 1.51)$

f) $x \in (0.1, 0.5)$

10. Answers may vary. For example, $f(x) = x^3 + 5x^2 + 2x - 8$ and $g(x) = 0$.

11. Answers may vary. For example, $f(x) = -x^2 + 25$ and $g(x) = -x + 5$.

12. $a \doteq 7$, $b \doteq 2$

13. Answers may vary. For example:

Perform the necessary algebraic operations to move all of the terms on the right side of the equation to the left side of the equation.

↓

Construct the function $f(x)$, such that $f(x)$ equals the left side of the equation.

↓

Graph the function $f(x)$.

↓

Determine the x-intercepts of the graph that fall within the interval provided, if applicable.

↓

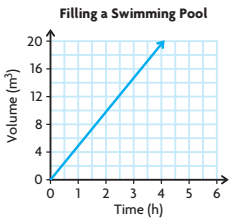
The x-intercepts of the graph are the solutions to the equation.

14. $x = 0 \pm 2n$, $x = -0.67 \pm 2n$ or $x = 0.62 \pm 2n$, where $n \in \mathrm{I}$

15. $x \in (2n, 2n + 1)$, where $n \in \mathrm{I}$

Lesson 9.7, pp. 569–574

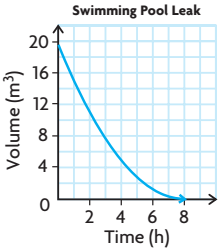
1. a)



b) $y = 6.25\pi\left(\frac{x}{4}\right)$

c) about 1.6 h

2. a) $y = \frac{6.25\pi}{64}(x - 8)^2$



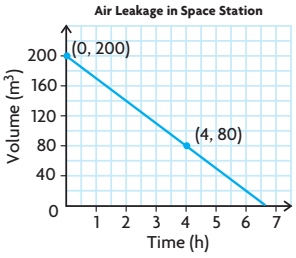
b) $V(t) = \frac{6.25\pi}{64}(t - 8)^2$

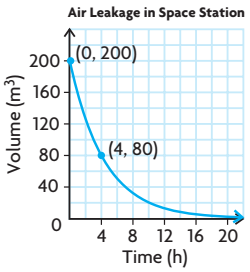
c) $V(2) \doteq 11\ \text{m}^3$

d) $-4.3\ \text{m}^3/\text{h}$

e) As time elapses, the pool is losing less water in the same amount of time.

3. a) Answers may vary. For example:

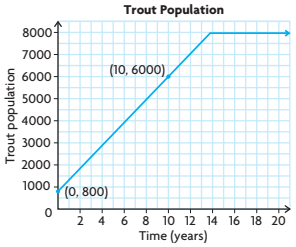




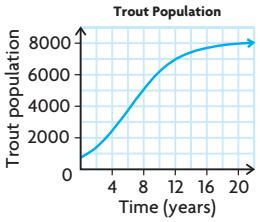
b) $V(t) = -30t + 200$; $t \doteq 6.7$

c) $V(t) = 200(0.795)^t$; $t \doteq 10$

4. a)



b) $P(t) = \frac{8000}{1 + 9(0.719)^t}$



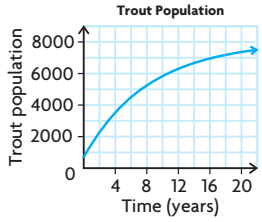
c) about 2349

d) 387.25 trout per year

5. a) the carrying capacity of the lake; 8000

b) Use (0, 800) and (10, 6000).
$a = 7200$, $b \doteq 0.88$

c)



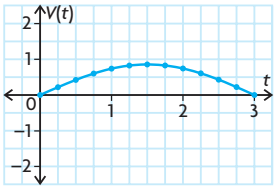
d) $P(4) \doteq 3682$

e) 720.5 trout per year

f) In the model in the previous problem, the carrying capacity of the lake is divided by a number that gets smaller and smaller, while in this model, a number that gets smaller and smaller is subtracted from the carrying capacity of the lake.

6. Answers may vary. For example, the first model more accurately calculates the current price of gasoline because prices are rising quickly.

7. a) $V(t) = 0.85\cos\left(\frac{\pi}{3}(t - 1.5)\right)$



b) The scatter plot and the graph are very close to being the same, but they are not exactly the same.

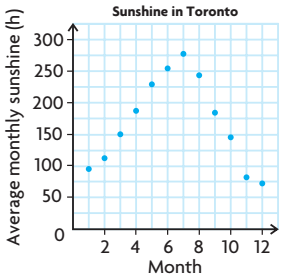
c) $V(6) = 0\ \text{L/s}$

d) From the graph, the rate of change appears to be at its smallest at $t = 1.5$ s.

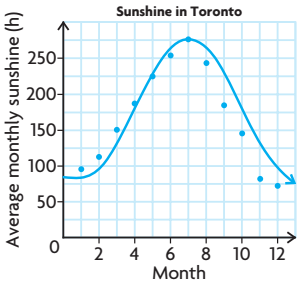
e) It is the maximum of the function.

f) From the graph, the rate of change appears to be greatest at $t = 0$ s.

8. a)



b)

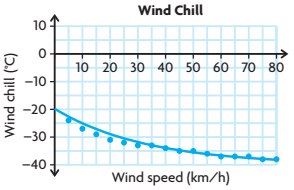


c) $S(t) = -97\cos\left(\frac{\pi}{6}(t - 1)\right) + 181$

d) From the model, the maximum will be at $t = 7$ and the minimum will be at $t = 1$.

e) It doesn't fit it perfectly, because, actually, the minimum is not at $t = 1$, but at $t = 12$.

9. a)



b) Answers may vary. For example, $C(s) = -38 + 14(0.97)^s$

c) $C(0) = -24\ ^\circ\text{C}$
$C(100) \doteq -37.3\ ^\circ\text{C}$
$C(200) \doteq -38\ ^\circ\text{C}$
These answers don't appear to be very reasonable, because the wind chill for a wind speed of 0 km/h should be $-20\ ^\circ\text{C}$, while the wind chills for wind speeds of 100 km/h and 200 km/h should be less than $-38\ ^\circ\text{C}$. The model only appears to be somewhat accurate for wind speeds of 10 to 70 km/h.

10. a) Answers will vary. For example, one polynomial model is $P(t) = 1.4t^2 + 3230$, while an exponential model is $P(t) = 3230(1.016)^t$. While neither model is perfect, it appears that the polynomial model fits the data better.

b) $P(155) = 1.4(155)^2 + 3230$
$\doteq 36\,865$
$P(155) = 3230(1.016)^{155} \doteq 37\,820$

c) A case could be made for either model. The polynomial model appears to fit the data better, but population growth is usually exponential.

d) According to the polynomial model, in 2000, the population was increasing at a rate of about 389 000 per year, while according to the exponential model, in 2000, the population was increasing at a rate of about 465 000 per year.

11. a) $P(t) = 3339.18(1.132\,25)^t$

b) They were introduced around the year 1924.

c) rate of growth $\doteq 2641$ rabbits per year

d) $P(65) \doteq 10\,712\,509.96$

12. a) $V(t) = 155.6 \sin (120\pi t + \frac{\pi}{2})$

b) $V(t) = 155.6 \cos (120\pi t)$

c) The cosine function was easier to determine. The cosine function is at its maximum when the argument is 0, so no horizontal translation was necessary.

13. a) Answers will vary. For example, a linear model is $P(t) = -9t + 400$, a quadratic model is $P(t) = \frac{23}{90}(t - 30)^2 + 170$, and an exponential model is $P(t) = 400(0.972)^t$.

The exponential model fits the data far better than the other two models.

b) $P(t) = -9t + 400$
$P(60) = -140$ kPa

$P(t) = \frac{23}{90}(t - 30)^2 + 170$,
$P(60) = 400$ kPa
$P(t) = 400(0.972)^t$, $P(60) \doteq 73$ kPa

c) The exponential model gives the most realistic answer, because it fits the data the best. Also, the pressure must be less than 170 kPa, but it cannot be negative.

14. As a population procreates, the population becomes larger, and thus, more and more organisms exist that can procreate some more. In other words, the act of procreating enables even more procreating in the future.

15. a) linear, quadratic, or exponential

b) linear or quadratic

c) exponential

16. a) $T(n) = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$

b) $47\,850 = \frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n$

So, $n \doteq 64.975$. So, it is not a tetrahedral number because n must be an integer.

17. a) $P(t) = 30.75(1.008\,418)^t$

b) In 2000, the growth rate of Canada was less than the growth rate of Ontario and Alberta.

Chapter Review, pp. 576–577

1. division

2. a) Shop 2

b) $S_{1+2} = t^3 + 1.6t^2 + 1200$

c) 1 473 600

d) The owner should close the first shop, because the sales are decreasing and will eventually reach zero.

3. a) $C(x) = 9.45x + 52\,000$

b) $I(x) = 15.8x$

c) $P(x) = 6.35x - 52\,000$

4. a) $12 \sin (7x)$

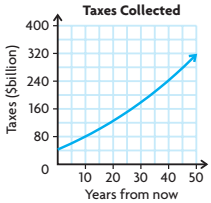
b) $9x^2$

c) $121x^2 - 49$

d) $2a^2b^{3x}$

5. a) $C \times A = 42\,750\,000\,000(1.01)^t + 3\,000\,000\,000t(1.01)^t$

b)



d) about $156 402 200 032.31

6. a) $\frac{21}{x}$

b) $\frac{1}{2x + 9}$

c) $\frac{\sqrt{x + 15}}{x + 15}$

d) $\frac{x^3}{2 \log x}$

7. a) $\{x \in \mathbf{R} \mid x \neq 0\}$

b) $\left\{x \in \mathbf{R} \mid x \neq 4, x \neq -\frac{9}{2}\right\}$

c) $\{x \in \mathbf{R} \mid x > -15\}$

d) $\{x \in \mathbf{R} \mid x > 0\}$

8. a) Domain of $f(x)$: $\{x \in \mathbf{R} \mid x > -1\}$
Range of $f(x)$: $\{y \in \mathbf{R} \mid y > 0\}$
Domain of $g(x)$: $\{x \in \mathbf{R}\}$
Range of $g(x)$: $\{y \in \mathbf{R} \mid y \geq 3\}$

b) $f(g(x)) = \frac{1}{\sqrt{x^2 + 4}}$

c) $g(f(x)) = \frac{3x + 4}{x + 1}$

d) $f(g(0)) = \frac{1}{2}$

e) $g(f(0)) = 4$

f) For $f(g(x))$: $\{x \in \mathbf{R}\}$
For $g(f(x))$: $\{x \in \mathbf{R} \mid x > -1\}$

9. a) $x - 6$

b) $x - 9$

c) $x - 12$

d) $x - 3(1 + n)$

10. a) $A(r) = \pi r^2$

b) $r(C) = \frac{C}{2\pi}$

c) $A(r(C)) = \frac{C^2}{4\pi}$

d) $\frac{C^2}{4\pi} \doteq 1.03$ m

11. $f(x) < g(x)$: $-1.2 < x < 0$ or $x > 1.2$
$f(x) = g(x)$: $x = -1.2$, 0, or 1.2
$f(x) > g(x)$: $x < -1.2$ or $0 < x < 1.2$

12. a) $x \doteq 4.0$

b) $x \doteq 2.0$

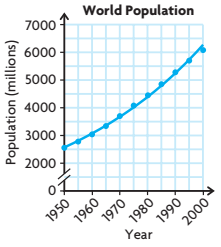
c) $x \doteq -0.8$

d) $x \doteq 0.7$

13. a) $P(t) = 600t - 1000$. The slope is the rate that the population is changing.

b) $P(t) = 617.6(1.26)^t$, 617.6 is the initial population and 1.26 represents the growth.

14. $P(t) = 2570.99(1.018)^t$



When $t = 13$, $P(t) = 3242$.
When $t = 23$, $P(t) = 3875$.
When $t = 90$, $P(t) = 12\,806$.

Chapter Self-Test, p. 578

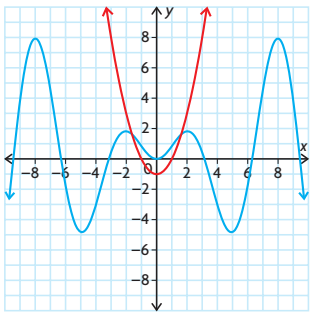
1. a) $A(r) = 4\pi r^2$

b) $r(V) = \sqrt[3]{\frac{3V}{4\pi}}$

c) $A(r(V)) = 4\pi\left(\frac{3V}{4\pi}\right)^{\frac{2}{3}}$

d) $4\pi\left(\frac{3(0.75)}{4\pi}\right)^{\frac{2}{3}} \doteq 4$ m^2

2.



From the graph, the solution is $-1.62 \leq x \leq 1.62$.

3. Answers may vary. For example, $g(x) = x^7$ and $h(x) = 2x + 3$, $g(x) = (x + 3)^7$ and $h(x) = 2x$

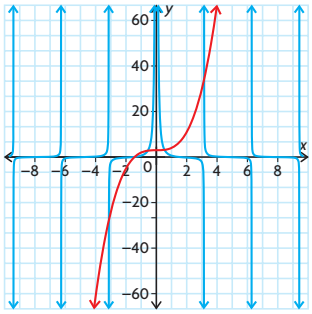
4. **a)** $N(n) = 1n^3 + 8n^2 + 40n + 400$
b) $N(3) = 619$

5. $(f \times g)(x) = 30x^3 + 405x^2 + 714x - 4785$

6. **a)** There is a horizontal asymptote of $y = 275$ cm. This is the maximum height this species will reach.
b) when $t \doteq 21.2$ months

7. $x = 4.5$ or 4500 items

8.



The solutions are $x = -3.1, -1.4, -0.6, 0.5$, or 3.2.

9. Division will turn it into a tangent function that is not sinusoidal.

Cumulative Review Chapters 7–9, pp. 580–583

1.	(d)	**10.**	(d)	**19.**	(c)	**28.**	(a)
2.	(b)	**11.**	(a)	**20.**	(d)	**29.**	(d)
3.	(a)	**12.**	(b)	**21.**	(b)	**30.**	(d)
4.	(a)	**13.**	(d)	**22.**	(a)	**31.**	(c)
5.	(d)	**14.**	(d)	**23.**	(c)	**32.**	(d)
6.	(c)	**15.**	(c)	**24.**	(c)	**33.**	(d)
7.	(d)	**16.**	(a)	**25.**	(c)	**34.**	(b)
8.	(b)	**17.**	(b)	**26.**	(b)		
9.	(c)	**18.**	(b)	**27.**	(a)		

35. 27° or 63°

36. **a)** Answers may vary. For example, Niagara: $P(x) = (414.8)(1.0044^x)$; Waterloo: $P(x) = (418.3)(1.0117^x)$
b) Answers may vary. For example, Niagara: 159 years; Waterloo: 60 years
c) Answers may vary. For example, Waterloo is growing faster. In 2025, the instantaneous rate of change for the population in Waterloo is about 6800 people/year, compared to about 2000 people/year for Niagara.

37. $m(t) = 30\,000 - 100t$,

$$a(t) = \frac{T}{30\,000 - 100t} - 10,$$

$$v(t) = -\frac{\log\left(1 - \frac{t}{300}\right)}{\log 2.72} - gt;$$

at $t = 0$, $\frac{T}{30\,000} - 10$ must be greater than 0 m/s^2, so T must be greater than 300 000 kg × m/s^2 (or 300 000 N)